Down to earth Celestial Navigation.

A few concepts to explain the
fundamentals of Celestial Navigation.

--With a sextant measure the angle from the horizon to a celestial body.

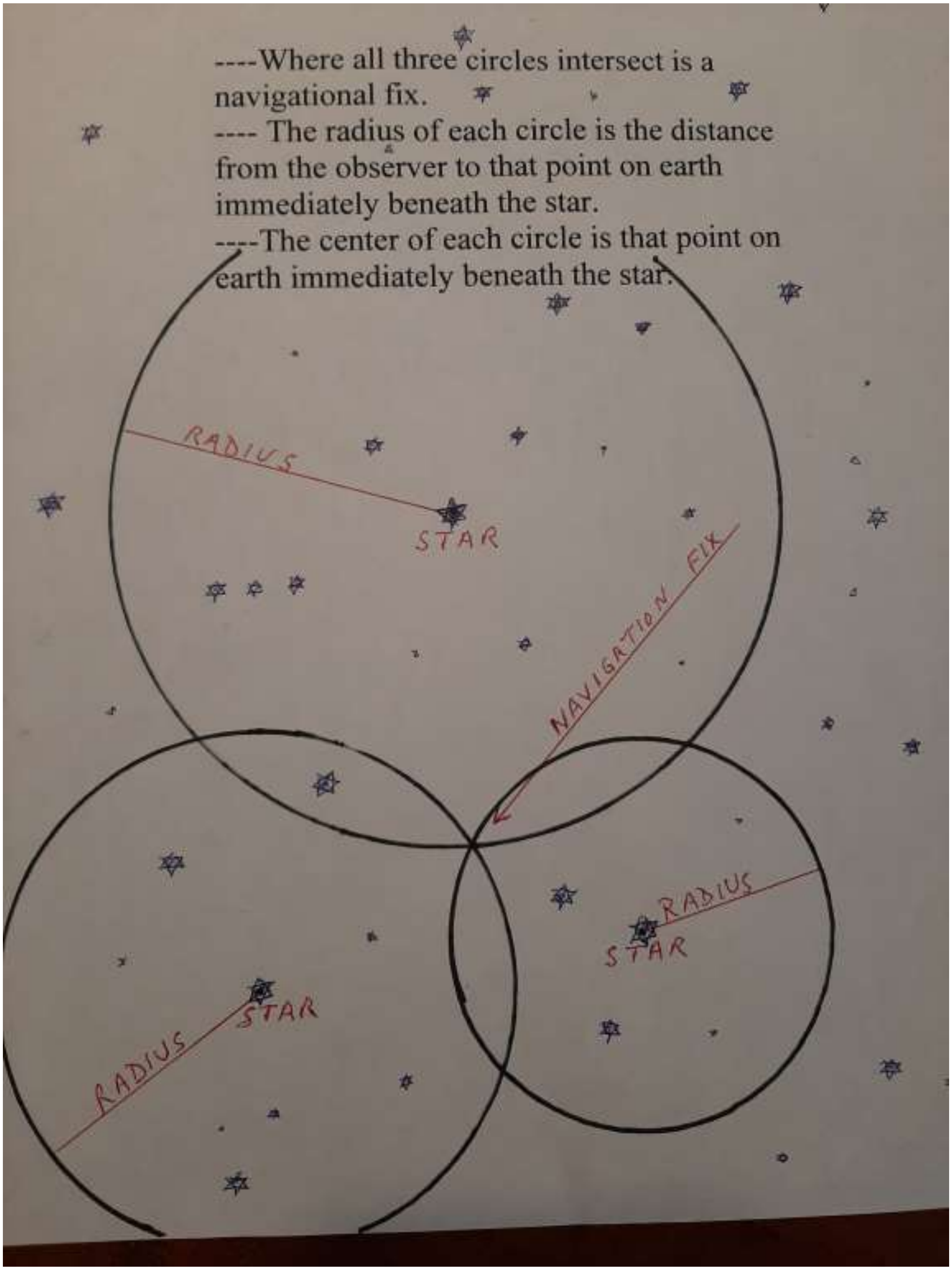
--The sextant angle (sextant altitude), corresponds to the distance from you, the observer, to the point on the earth directly beneath that celestial body; -- the Ground Position (GP) of that celestial body at that particular time. A specific sextant altitude in degrees relates to a specific distance in nautical miles on the surface of the earth.

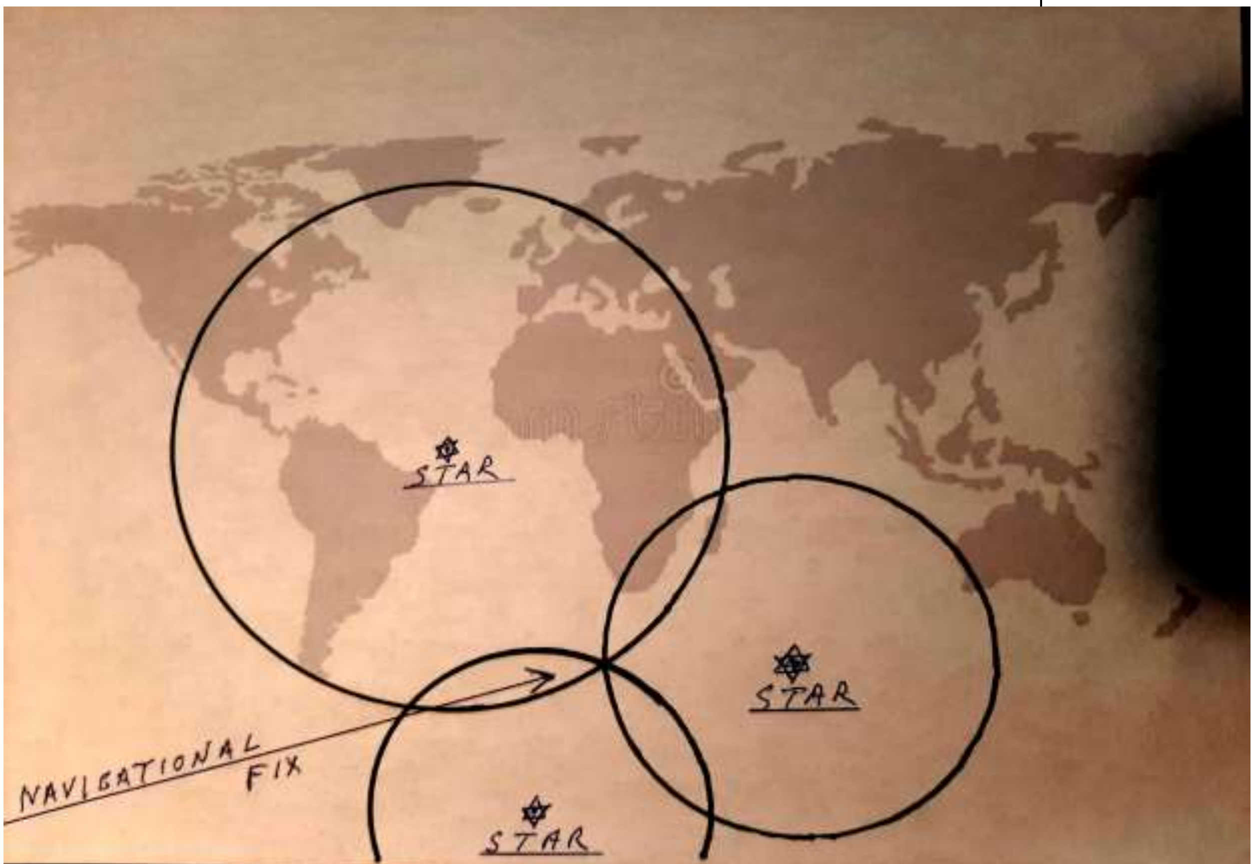
--The Nautical and Air almanacs tabulate the coordinates of the Ground Position (GP) of each of the celestial bodies used in navigation, and can be interpolated to the nearest second of time.

--The sextant altitude places the observer somewhere on a circle surrounding that point on earth directly beneath the celestial body. Anyone anywhere on the circumference of that circle would get the same sextant altitude for that celestial body at that same time. Anyone, anywhere on the circumference of that circle would be the same distance from the ground position of that celestial body. The radius of the circle is the distance between the observer and that point on earth directly beneath the celestial body; -- the Ground Position (GP) of that celestial body.

--Take sightings on two or more celestial bodies and where these circles intersect is the observer's position,-- a "Navigational Fix.

Some diagrams to illustrate these fundamentals


----Where all three circles intersect is a navigational fix.
---- The radius of each circle is the distance from the observer to that point on earth immediately beneath the star.
----The center of each circle is that point on earth immediately beneath the star.
RADIUS
STAR
NAVIGATION FIX
RADIUS
STAR
RADIUS
STAR


STAR
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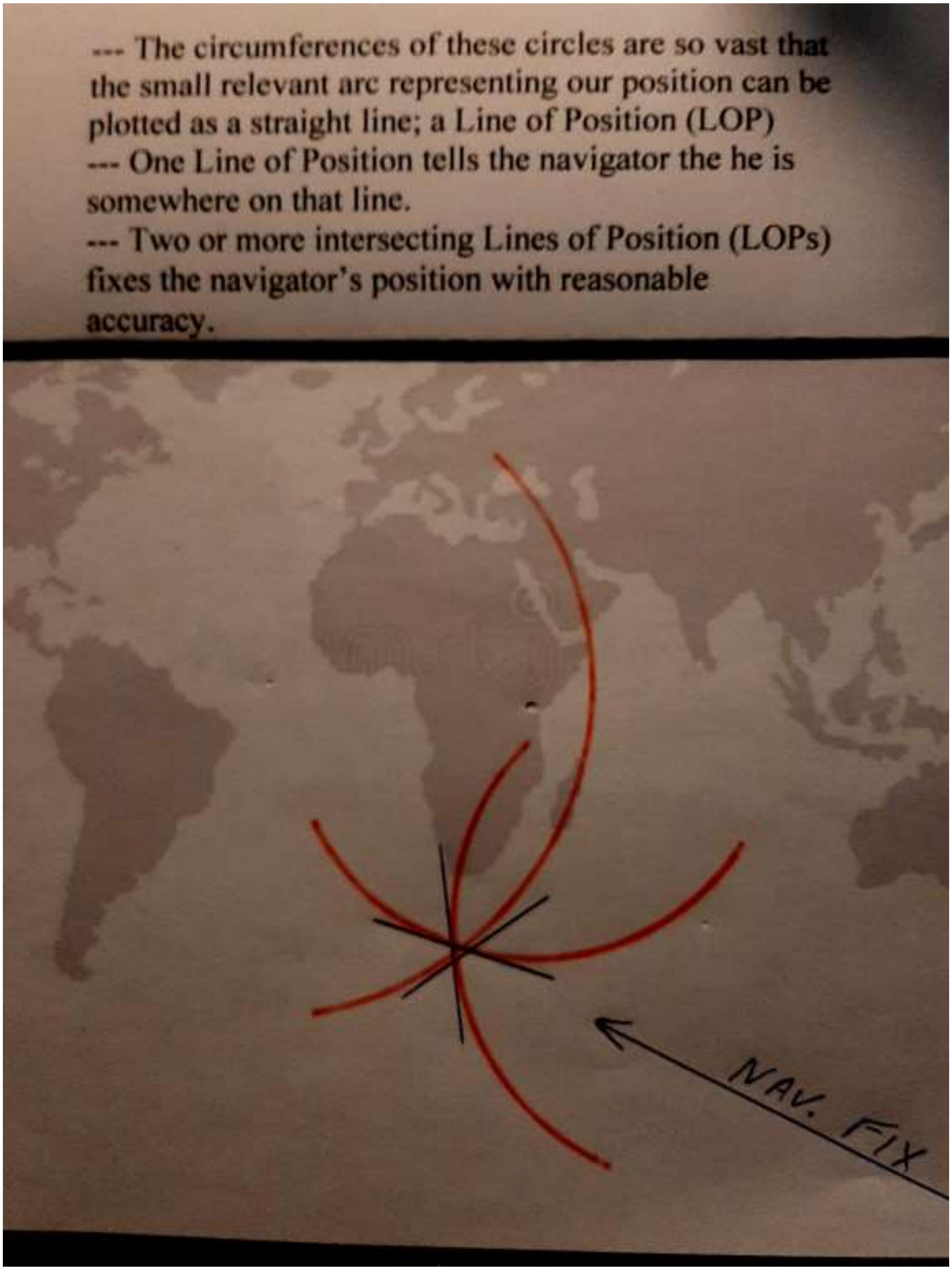
STAR
NAVIGATIONAL FIX



--- The circumferences of these circles are so vast that the small relevant arc representing our position can be plotted as a straight line; a Line of Position (LOP)
--- One Line of Position tells the navigator the he is somewhere on that line.
--- Two or more intersecting Lines of Position (LOPs) fixes the navigator's position with reasonable accuracy.

A three position-line fix.

As you can see. It's all about circles.

Keep these simple diagrams in mind. They are the basis of Celestial Navigation.

The intersection of only two such circles will still provide a Navigational Fix. As you will see in the diagrams however, these circles intersect in two places. Only one of these intersections is the navigator’s correct position. Rarely, if ever, is this a cause of any ambiguity, as these two points of intersection are usually separated by hundreds, and more often by thousands of miles. Also the mechanism, (described later), for plotting a line of position resolves any doubt. In any event taking a third sighting definitively resolves any possible ambiguity.

In Celestial Navigation, pin point accuracy, as perhaps intimated by the term “Navigational Fix” is not attainable.

A Navigational Fix is sometimes also referred to, and possibly more correctly, as “Observed Position.”

However in this tutorial I will continue to use the term “Navigational Fix”.

As you will see in the above diagrams, the basic concept in Celestial Navigation is quite simple.

However, the devil and the details enter the picture when we

attempt to plot on our chart these circles that can have radii of thousands of miles, and circumferences that pass through different hemispheres, different oceans, and different continents. (Diagram Page 4).

An explanation of the **"Intercept method",** used to plot a line of position commences on page 47.

This is a mechanism that enables us to plot a line of position on our chart by using our dead reckoned position (or an assumed position close to it) as a reference point.

In order to get a good initial understanding of the topic without becoming too bogged down in detail, perhaps first read the tutorial omitting those parts of the text (Pages 66-86) in red.

Those parts of the text in red are used to provide a more in depth explanation of some of the aspects of celestial navigation, as well as containing additional diagrams and explanations to help further clarify the topic. They also deal with the math used in plotting a line of position. (Pages 76 to 86.)

This Tutorial is in four parts:

………………………………………………………………

Some preliminary comments:

Celestial Navigation has its own language. However, despite the at times, unfamiliar terminology you will be relieved to find that, as shown in the initial diagrams, the basic concept of Celestial Navigation still remains eminently simple.

Some of the concepts and terminology you will be familiar with. However some may be new and at first intimidating. Do not be put off by this. In the ***Terminology*** *section below one has to start with a bald litany of what may be unfamiliar terms, but with the help of the diagrams that are interspersed throughout the text these terms become more intelligible. As you become more familiar and more at ease with them, the concepts and terminology will make sense, and are, in any event, much more easily understood in the context of their usage. If you have difficulty initially with some of the terminology it will* *become clearer as it is introduced in context and with explanatory diagrams as the tutorial progresses.*

One approach might be to quickly scan through the terminology section initially in order to gain some working knowledge of the terms, subsequently referring back to it if, and as, needed.

Latitude by the noon sun, and Longitude by chronometer, are simpler and special applications of celestial navigation, and are discussed towards the end of the tutorial.

I have attempted to keep the diagrams simple and uncluttered.

The diagrams used in this tutorial will be no more complex than those you have already seen, on pages 3, 4, and 5.

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Terminology

--True North The point in the Northern Hemisphere where the earth's axis meets its surface. The earth revolves eastward around this axis. Also known as the North Geographic Pole, and located in the Arctic Ocean.

--Polaris The earth's axis points almost directly at the North Star, Polaris. Polaris, however, is currently about three quarters of a degree, -- 45 minutes of arc, -- away from being directly in line with the earth's axis, so that in effect, as the earth rotates on its axis Polaris describes a small circle around the North Celestial Pole, and this has to be taken into account when finding latitude by Polaris.

--South Geographic Pole. There is no easily visible star to mark the South Pole. A line between two stars of the Southern Cross, Gacrux and Acrux, can be extended to extrapolate the position of the South Celestial Pole.

--Celestial Body. The Celestial bodies used in navigation are: The Sun, together with 57 other navigational stars, the Moon, and four 4 planets, -- Venus, Mars, Jupiter and Saturn. One other star, Polaris, is also used in navigation, and because of its unique position is of special significance.

In celestial navigation it is convenient to imagine a pre-Copernicus universe in which celestial bodies move in a westerly direction around a stationary earth.

--Latitude. Angular distance north or south of the Equator. The angle is a measurement from the center of the earth.
One degree, as measured from the center of the earth, represents 60 nautical miles (NM) on the surface of the earth. One minute of arc, (one sixtieth of a degree) represents one nautical mile (NM).
Equator represents zero degrees latitude. Either Pole represents 90 degrees latitude, North and South.

-- Great Circle Any circle on the surface of the earth whose center passes through the center of the earth.
Any circle that divides the earth into two equal halves.
Any angle subtended at the center of the earth represents a distance on the surface of the earth, -- one degree representing 60 nautical miles, and this distance on the surface of the earth being part of a great circle.
All meridians of longitude are part of a great circle.

No parallel of latitude, other than the equator, is a great circle.
A great circle route is the shortest distance between any two points on the earth's surface.

--Longitude. An angular measurement east and west of the Prime Meridian. Meridians of Longitude are imaginary lines running north and south and meeting at the poles.
Meridians of Longitude are measured 180 degrees east and 180 degrees west from the Prime Meridian.

--**Prime Meridian**. The reference meridian representing zero degrees of longitude, that runs through the Greenwich Observatory in England.
While the equator, mid way between the Poles, is an obvious North-South reference line, no such obvious East-West reference presents itself. From ancient times it was understood that some empirical reference was needed as an East-West coordinate reference. Throughout history many sites for the Prime Meridian had been considered. Hipparchus, a Greek astronomer who lived in the second century b.c., suggested the island of Rhodes as a candidate. In more recent times, Paris, the Canary Islands, the Azores, the Great Pyramid of Giza, and the American Whitehouse were all in the running.
In 1884 a group of countries got together and voted, almost unanimously, to select the meridian that runs through Greenwich, England, as the Prime Meridian.
France had lobbied for the Prime Meridian to run through Paris, and huffily abstained from the voting. However, Air France and the French Navy have long since bowed to the

inevitable, and with a Gallic shrug of acceptance, have acknowledged a “fait accompli”!

--Hs. Initial raw sextant reading, prior to any corrections.

--Ho. Actual true sextant altitude, after applying various corrections, -- for height of eye, for refraction, and others.

--Hc. Computed altitude. Not taken with the sextant, but mathematically *computed* from an assumed or reference position on the chart, using certain known inputs.

--A.P. Assumed Position. A selected reference position on our chart, at or near to the dead reckoned position, from which a *computed* altitude (Hc) is *calculated*. This assumed position becomes a reference point from which it is then possible to plot our actual line of position on our chart.

Some may take exception to the wording “Assumed Position” in that the navigator is not under the assumption that this is his or her position. A more accurate terminology perhaps would be “reference position”

--**Dead Reckoning**. Estimating one’s present position by the application of elapsed time to one’s course and speed from last known position, taking into consideration also effects of wind and tide if known, -- and if applicable. (If tide becomes a consideration in air navigation, perhaps altitude would be a more pressing one!

--Line of Position. (LOP) A line, plotted on our chart on some part of which we are located. (In celestial navigation actually a small part of the arc of a circle of equal altitude, represented as a straight line). (See diagrams on Pages 4, and 44). Where two or more such lines intersect is a Navigational Fix.

--Navigational Fix. The plotting of one's geographical position. Traditionally referenced in Latitude and Longitude on a navigational chart.

--Circle of equal altitude. Also referred to as "Circle of Position." A circle surrounding the ground position of a celestial body, the radius of which is the distance between the observer and the ground position of that celestial body, - by definition, Zenith Distance. (Diagrams pages 3 and 31) The observer is somewhere on the circumference of this circle, which relates to a specific sextant altitude (Ho), and anyone anywhere on the circumference of this circle would get the same sextant altitude (Ho) for the same celestial body at that same time.

--Zenith Distance (ZD) Zenith distance is the complimentary angle of the sextant altitude (Ho).

ZD= 90 – Sextant Altitude (Ho)

Zenith distance is both an angle in degrees, and a distance in nautical miles. Multiplying the zenith distance (ZD) angle by 60 gives us zenith distance in nautical miles.
On the surface of the earth it is the distance from the observer to the Ground Position (G.P). of the observed body. It is also part of a great circle on the earth's surface.

In the diagrams at the beginning of this tutorial Zenith Distance is the radii of the circles in those diagrams, i.e. the

great circle distance on earth's curved surface from observer to Ground Point (GP) of celestial body.
Zenith Distance is an important concept in Celestial Navigation, and its relationship to sextant altitude is demonstrated also in the diagrams on pages 33 and 39. The exact position for a specific time, of any of these celestial bodies used in navigation is to be found tabulated in the Nautical Almanac to an accuracy of one second of time.

Coordinates of Ground Position (G.P). of celestial bodies used in navigation, are expressed in Declination (Latitude) and Greenwich Hour Angle.(GHA) (Longitude). The accuracy for these positions is to within 6 seconds of arc, (one six hundredths of a degree, or around 600 feet.)

--G.P. **Ground Position**. The position on earth's surface directly beneath a celestial body. A line from the celestial body to the center of the earth would intersect the earth's surface at the ground position of that particular celestial body. When standing at the Ground Position of a celestial body, the celestial body is directly overhead the observer, i.e. at the observer's zenith.

Every celestial body has an exact position directly beneath it on the surface of the earth. However, as the earth is rotating eastward, these positions are constantly changing, moving westward, second by second.

Due to this eastward rotation of the earth on its axis, celestial bodies move westward over the surface of the earth at an average speed of 360 degrees in 24 hours, which is one degree every 4 minutes, or one nautical mile every 4 seconds at the equator.

--**Zenith**. The point on the celestial sphere directly above the observer's head.

--**Declination**. The earth's coordinates of Latitude and Longitude are projected onto a celestial sphere. Declination equates to Latitude, Greenwich Hour Angle (GHA) equates to Longitude in reference to celestial bodies on this celestial sphere, and to their ground positions (GP) on earth.

Hour angle. Since the earth rotates beneath the sun at an average (mean) rate of 15 degrees every hour, this angular measurement can also be expressed as an hourly measurement, -- or as an hour angle.

--GHA. Greenwich Hour Angle. Equates to Longitude in reference to celestial bodies, but unlike Longitude, (which is measured 180 degrees east and west from the Prime Meridian), GHA is measured westward from the Prime Meridian through 360 degrees all the way around the globe.

--LHA. Local Hour Angle. The angle at the Pole between the longitude of the observer and the longitude (Greenwich Hour Angle. GHA) of the ground position of the celestial body. LHA is measured westward from observer's position.
LHA = GHA – West Longitude.
LHA = GHA + East Longitude.

--**<u>Celestial Sphere</u>**. An imaginary sphere surrounding the earth on the inside of which the celestial bodies are located, and onto which the earth's grid system of coordinates can be projected, – a planetarium view of the heavens. In Celestial Navigation it is useful to imagine a pre-Copernicus universe in which this celestial sphere rotates in a westerly direction around a stationary earth.

<u>--Sidereal Hour Angle</u>. The angular measurement of a star westward from a reference point on the celestial sphere. This reference point is know as the First Point of Aries.

Because of their vast distances from us all of the stars appear in a fixed unchanging position relative to each other.

If the nautical almanac were to tabulate the Greenwich Hour Angle (GHA) (Longitude) of each of the 57 navigational stars interpolated for each second of time, (as it does for the sun moon and planets) it would run into many volumes. Since the stars appear in a fixed unchanging position relative to each other it is only necessary to tabulate the Greenwich Hour Angle (GHA) of one fixed point among the stars – First Point of Aries -- and then merely tabulate each star's fixed position relative to Aries. In this way it is unnecessary to tabulate the Greenwich Hour Angle (GHA) of each individual star.

The angular distance of each star measured *westward* from this fixed point of Aries is each star's Sidereal Hour Angle (SHA)

The Sidereal Hour Angle (SHA) for each of the 57 navigational stars is listed in the nautical almanac, taking up only a portion of a page.

To find the Greenwich Hour Angle (GHA) of a listed star one merely finds the Greenwich Hour Angle (GHA) of Aries for any second of time, and add to this the Sidereal Hour Angle (SHA) of the star.

--First Point of Aries The First Point of Aries is the position of the sun on the celestial sphere at the time of the Vernal Equinox. This is a reference point on the celestial sphere from which the Sidereal Hour Angle (SHA) of a star is measured westward. While The First point of Aries is not a star, but an empirical reference point in space, it is helpful to imagine it as a star, with all other stars measured westward from a celestial meridian which passes through it.

While for practical purposes we regard the First Point of Aries as a fixed point in space, its position in space does actually change slowly, -- one degree every 72 years. This change in position is due to the slow precession of the earth's axis.

[--**Right Ascension (RA)]**This term is added here for interest only. RA is used in astronomy, and in star charts, together with declination, as a star's coordinate reference, but is not used in celestial navigation. Right ascension is similar to Sidereal Hour Angle (SHA) with the difference being that Right Ascension (RA) is measured *eastward* from the first point of Aries, and right ascension is commonly measured in hours, minutes and seconds of time. SHA is measured *westward* from Aries, and measured in degrees, arc minutes and decimals of arc minutes.

--Time There are many ways to measure time depending on the reference criteria. In celestial navigation we deal mainly with three types of time, each of which is referenced to the sun:

(1) Greenwich Mean Time,

(2) Local Apparent Time. "Apparent" in this context means true time by the sun.

(3) Zone time.

--GMT. Time measured at Greenwich on the zero degree of longitude. It is 1200 GMT when the *mean* sun is on the Greenwich zero meridian. The sun actually has a varying speed of rotation around the earth, moving faster around the earth on some days slower on others. (Again pre-Copernicus!) Greenwich Mean Time averages out these discrepancies so that the sun is assumed to arrive over the Prime Greenwich Meridian exactly at the same time –1200, Noon -- each day. The discrepancies between the *mean* (average, not nasty!) sun and the *apparent* (true) sun are reconciled by what is known as the **"*Equation of Time*" (EoT).**

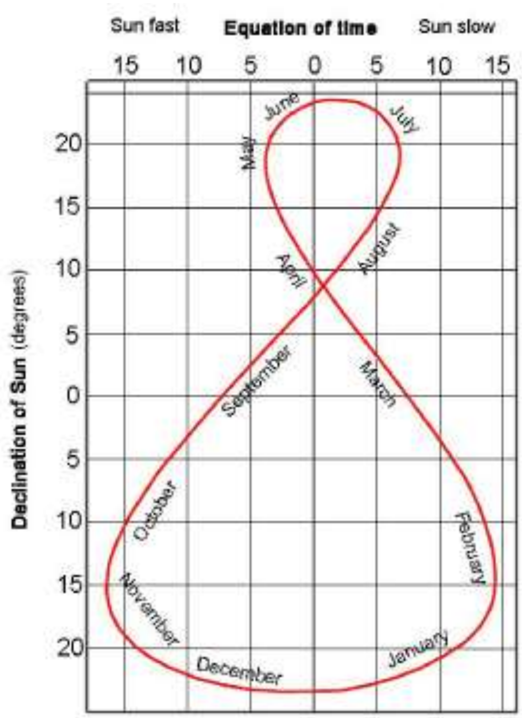
Two of the reasons that the apparent (true) sun does not accurately appear over the Prime Meridian at noon every twenty four are:

(1) Due to the earth's elliptical orbit in its annual trip around the sun.

(2) Due to the tilt of the earth's axis.

The figure-of-eight like diagram, below, called an "analemma" that one sees on some sundials, is a graphical representation of the Equation of Time. (EoT) It shows the relationship of Mean Solar Time to Apparent (True) Solar Time throughout the year.

The two graphs show the maximal discrepancies between the true sun and the mean sun as occurring in February and November- the true sun arriving close to15 minutes late at Greenwich in February, and arriving close to fifteen and one half minutes early at Greenwich in November.



Greenwich Mean Time is also referred to as --Universal Time Coordinated. (U.T. or U.T.C.)

In the military referred to as Zulu time, (Z). Greenwich mean time (GMT) is based on astronomical observations of the sun, whereas Universal Time (UT) is based on atomic clocks. In celestial navigation they are the same for all practical purposes, and I will use the (formerly) commonly used term, Greenwich Mean Time, rather than Universal Time.

--**Local Apparent Time**. Local apparent time is the time shown on a sun dial.

The word "Apparent" in Celestial Navigation is an unfortunate choice of words as the word "apparent" can have two somewhat contradictory meanings.

It can mean "seemingly true," or it can mean " obviously true." In the context of celestial navigation "apparent" means "obviously true"

Local Apparent Noon (LAN) is the time when the apparent (true) sun crosses the observer's meridian of longitude and when, the apparent sun is either due north or due south or directly overhead the observer. It is that time of day when the observer at his position on earth sees the apparent sun at its highest point in the sky. It is noon on a sundial.

--**Zone Time** (Diagram page 102)

The earth is assumed to rotate eastward beneath the sun at an average rate of 15 degrees per hour.

In zone time the earth is divided into 24 time zones, each zone spanning (in most instances) 15 degrees of longitude, and each 15 degrees of Longitude representing one hour.

The zone time for each 15 degrees of longitude is referenced to the central meridian of that particular time zone.

All places within a specific 15 degree time zone keep the same time on their watches.

This central meridian of each time zone is referenced East or West from the prime Greenwich meridian, i.e. from 0 degrees longitude.

Time zone zero, the reference time zone at Greenwich, spans seven and one half degrees either side of the zero degrees Longitude line -- The Prime Meridian,

Time zone 12 spans seven and one half degrees either side of Longitude 180 degrees, the Antimeridian.

Time zones to the east of Greenwich are specified by the number of hours they are *ahead* of Greenwich.

Time zone "plus 5" would be 75 degrees (15 degrees x 5) east longitude and 5 hours *ahead* of Greenwich Mean Time (GMT).

When it is 1200 noon at Greenwich it would be 1700 at time zone "plus 5". At zone plus 5 their noon sun has come and gone.

Time zone "minus 6" would be 90 degrees (15 degrees x 6) west longitude and 6 hours *behind* Greenwich Mean Time.

When it is 1200 noon at Greenwich it would be 0600 at time zone “minus 6” At zone minus 6 their noon sun has not yet arrived.

To obtain Greenwich Mean Time (GMT) we subtract our zone number from our zone time (east longitude), or add our zone number to our zone time (west longitude).

It should be noted that there are many differing methods of naming time zones, and also for various pragmatic reasons time zones do not always conform to exact 15 degree boundaries.

The International Date Line is 180 degrees removed from the Prime Meridian and follows mostly the Antimeridian, passing through the largely empty Pacific, and Antarctica. In places it deviates from the 180th Degree of Longitude to accommodate political boundaries. Crossing the date line east to west we gain a day, west to east we lose a day.

--Military time. A 24 hour time reference, from midnight to midnight. 0001 is one minute after midnight, Noon is 1200 hours, 1.00pm is 1300 hours and so on to midnight which is 2400 hours. When referenced to Greenwich Mean Time, Military time, is designated with the postscript, Z for Zulu, as in 1400Z, -- equals 1400 Greenwich Mean Time (GMT)

--**Equation of Time**. (EoT) Because the speed of the apparent sun around the earth (pre-Copernicus concept!) varies slightly from day to day the apparent sun is not always directly over the Greenwich Meridian at noon every day. (Only on four days throughout the year is the apparent sun actually over the Greenwich Meridian at 1200 noon.) Some days the apparent sun is running slow, by as much as 30 seconds, some days its running fast by as much as 20 seconds. These daily discrepancies accumulate, so that they add up to around 15 minute discrepancies in February and November. For this reason and for conformity we have a *mean* sun which averages out these discrepancies, and this mean sun (averaged sun) is assumed to appear faithfully over the Prime Meridian at 1200, noon each day. This mean sun is assumed to travel at a uniform daily speed around the earth. From this mean sun we derive Greenwich Mean (average) Time. By the application of the Equation of Time we can reconcile the apparent (true) sun with the mean sun and vice versa.

Eot = Apparent sun – Mean sun.

If Apparent Solar time is ahead of Mean Solar time the sign is positive, if behind the sign is negative.

-- **True Bearing.** An angular direction, measured clockwise through 360 degrees, using True North as the 0 degree reference. 45 degrees True would be written as;

045T.

-- Precession, and the North Star. Today the earth's axis points almost directly (within three quarters of one degree) at Polaris. However, in the 1500s, in the time of Columbus, Polaris, while still regarded as the North Star, was more than 3 full degrees off the earth's axis. Additionally, due to precession, (wobble) of the earth's axis, this axis will gradually move away from the vicinity of Polaris, over thousands of years, and other stars will eventually become the North Star.

5,000 years ago, in the time of the early Egyptian Pharaohs, the star Thuban in the Draco constellation was their North Star. Today Thuban is 25 degrees off the earth's axis!

It is often theorized that the Pyramids were built to align with certain stars. If so, they certainly no longer remain in such alignment. Over the centuries, due to precession, they would drift into, and out of, alignment with various other celestial bodies. While the orientation of the pyramids with respect to the stars may change, their terrestrial orientation with respect to the earth's North Geographic Pole will remain, to all intents and purposes, constant.

In another 5,000 Alderamin in the constellation Cepheus will be the North Star and in another 12,000 years it will be Vega in the Summer Triangle.

So when Shakespeare had Julius Caesar say;

"But I am constant as the Northern Star." the bard was placing the emperor on wobbly ground!

While Shakespeare's "*Northern Star"* would have been Polaris, Caesar's would not have been. Caesar's North Star would have been the star Kochab. Today Kochab is 16 degrees off the earth's axis!

This precession of the earth's axis has a cyclical period of around 26,000 years, so it is not something of immediate concern to the navigator!

A Few Comments

In many instances, the text is repetitive. This is in the belief that important concepts bear repeating, and also in the hope that by repetition and by describing the same thing in slightly differing contexts it may help with better understanding. The reader has the option of either quickly skimming over, or bypassing altogether, the repetitive portions.

I have attempted to frequently re-explain the terminology with each usage.

I have attempted, in most instances, to use abbreviations only after the full textual terminology in the hope that the reader will become familiar with these abbreviations, rather than being constantly confused by them.

As relevant details are added to the overall picture, remain focused on the basic principle, i.e. a specific sextant altitude places the observer somewhere on a specific **"circle of equal altitude"** around the ground position of the celestial body. Take several sextant altitudes on several different bodies and where these circles intersect is your position.(Pages 3, 4, and 5.)

As you will see, however, while the overall principle remains simple, plotting these circles of (often) huge circumferences is where the details, -- and the math emerge.

In order that the reader will fully understand the basics, much of this tutorial is taken up in preliminary explanation of the concepts applied in Celestial Navigation, before we get to the actual plotting of a line of position, which commences on Page 47.

When the reader has an overall understanding of how celestial navigation works this essentially simple process becomes demystified, and enjoyable.

Celestial bodies are at such vast distances from the earth that their light can be regarded as arriving at earth in parallel rays. However when using the Moon, Mars or Venus in celestial navigation one needs to apply an added parallax correction due to their “closeness” to earth. With respect to the moon this parallax correction can be considerable. This concept of light from distant celestial bodies arriving in parallel rays is important in Celestial Navigation and is depicted in the diagrams explaining Zenith Distance (ZD) on pages 37 and 39.

However some other diagrams in the text, for simplicity of illustration, may show such rays angling outwards from the celestial body towards the earth. These conceptual diagrams, as on pages 31, 33,and 54 while useful in illustrating some particular aspect of celestial navigation, are not essentially correct, as in some instances, if solved mathematically, they would place the celestial body a few thousand miles from earth, rather than light years from it. The mathematician, indignant at such cavalier manipulation of geometry, and the astronomer equally indignant at the cavalier manipulation of the heavens, will both be reassured by the diagrams on pages 37 and 39!

--One degree of latitude equals 60 nautical miles. One degree at the center of the earth represents 60 nautical miles on the surface of the earth. One degree is further divided into 60 minutes (of arc) Thus one minute (of arc) equals one nautical mile. Minutes of arc are further divided into seconds of arc. There are 60 seconds of arc in one minute of arc. More commonly in celestial navigation we speak of degrees, minutes and tenths of minutes of arc, rather than degrees minutes and seconds of arc.

--Declination (Latitude) north-south apparent movement of a celestial body changes relatively slowly. For instance the sun’s change in declination (passage between the Summer and Winter Solstices) averages about one degree every four days.

The above however is an average, as the sun’s declination changes more rapidly close to the equator and more slowly approaching the solstices.

The change in declination of any star, other than the sun, due to their vast distances from us, is negligible.

-- Greenwich Hour Angle (GHA) (Longitude) of any celestial body changes rapidly throughout each day due to the eastward rotation of the earth on its axis. The east-west ground position of the sun changes on average, 360 degrees every 24 hours. This equates to one degree every four minutes which equates to one nautical mile every four seconds at the equator. Thus the importance of accurate time, when taking sextant altitudes.

--A specific sextant altitude relates to a specific distance between the observer and the ground position of the celestial body, --Zenith Distance (ZD),-- and results in a specific “**Circle of equal altitude”** around that body**, --(**Diagrams Pages 3, 4, 31, 33, 37.)**.**

The observer can be anywhere on the circumference of that circle of equal altitude, that has the ground position of the celestial body as its centre, and the distance between the observer and the ground position of the observed body as its radius, i.e. Zenith Distance (ZD). Any observer situated anywhere on the circumference of that circle would get the same sextant altitude on that same celestial body for that same time.

Sextant Altitude (Ho) relates to distance in the following manner:

[90 minus sextant altitude (Ho)]= Zenith Distance angle.(ZD).

Zenith distance angle (ZD) x 60 = distance in nautical miles, between the observer and the ground position of the celestial body.

Below are three important concepts in celestial navigation :

If you scroll down and correlate the text of each concept with the accompanying diagram below on page 31, you will see that all three concepts are eminently simple, as illustrated also in diagrams on page 33.

Again, some of these diagrams while conceptual, are on occasion, useful in explaining some particular aspect of celestial navigation.

(1) **Zenith Distance (ZD)**

--It is the distance from observer to that position on the earth directly beneath a celestial body.

--It is part of a great circle on the earth's surface. (Diagram Page 31)

--It is the curved radius of the circle of equal altitude.

--It is the complement of the sextant altitude (Ho)

--It is 90 degrees – Sextant angle (Ho).

--It represents both an angle and a distance.

-- (90 – Ho) x 60 = Zenith Distance in nautical miles.

(2) **Circle of equal altitude**

--It is a circle surrounding the Ground Position (GP) of a celestial body, the radius of which is the Zenith Distance, (ZD) i.e. distance of Observer to Ground Position of celestial body.

-- It is a circle anywhere on the circumference of which any observer would obtain the same sextant altitude for the same celestial body at that same time.

-- It is a circle somewhere on the circumference of which the observer is situated.

--It is a circle the size of which varies according to the sextant altitude (Ho) The smaller the sextant angle (Ho) the larger the zenith distance (radius) and consequently the larger the circle of equal altitude.(Diagrams Page 33).

-- It is a circle usually of such vast circumference that a small part of its circumference can be charted as a straight line,-- a Line Of Position,-- an (LOP). (Page 5).

(3) **Ground Position (GP)**

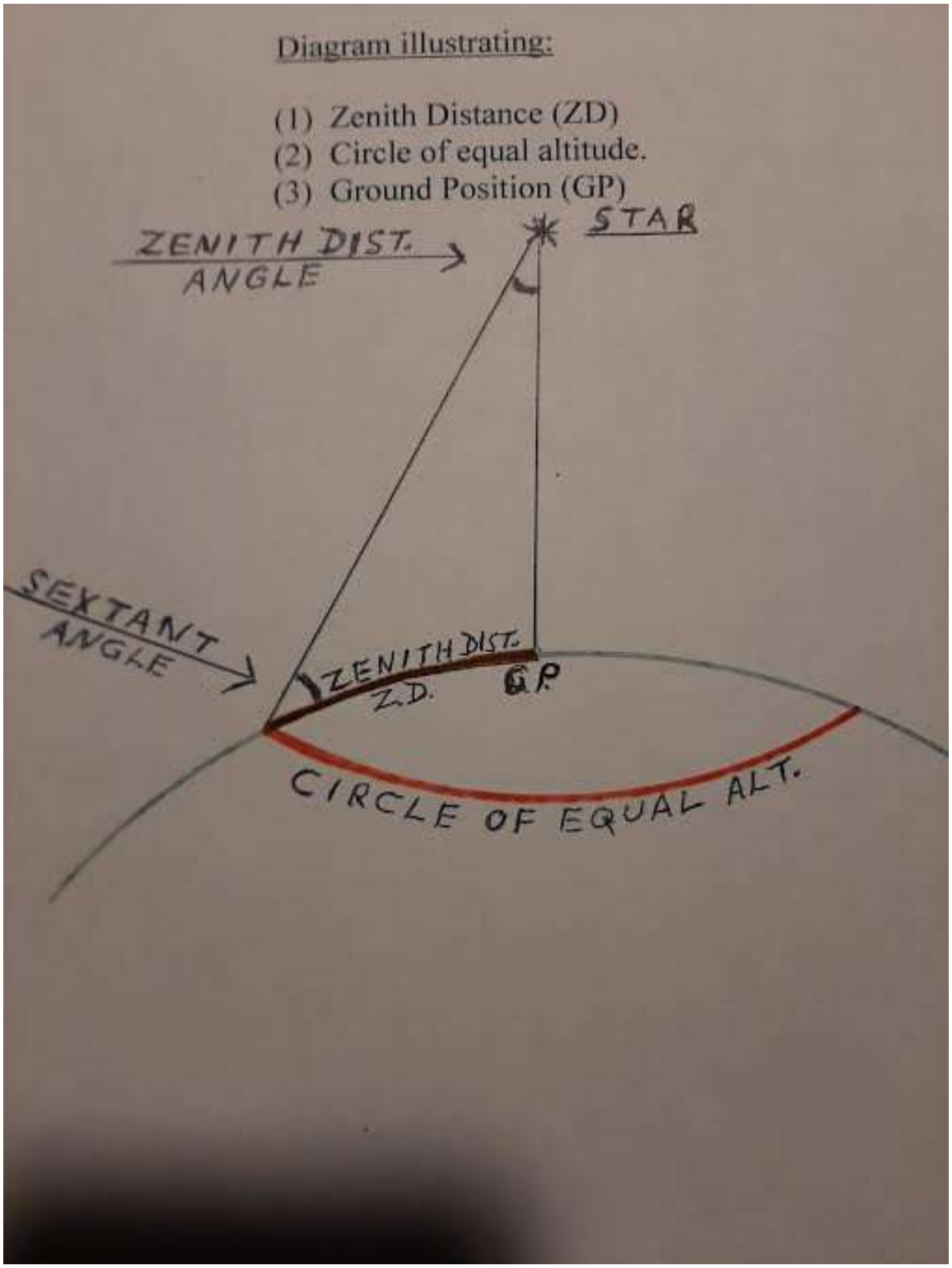
--It is the position on earth directly beneath a celestial body for any given time.

-- It is the centre of a circle of equal altitude for any specific sextant altitude, for any celestial body.

-- The distance from the observer to the ground position (GP) is the Zenith Distance (ZD). (The radius of the circle.)

-- The Ground Position (GP) of any celestial body changes constantly throughout the 24 hour period.

-- The Ground Positions (GP) of all of the celestial bodies used in navigation are listed in the Nautical Almanac and can be interpolated for each second of each day.



Diagrams of Circles of equal Altitude

In the next two diagrams below, the distance from observer to Ground Position (G.P.) of celestial body, Zenith Distance, (ZD) is again accentuated in heavy ink.

Again these two diagrams, if solved mathematically could place the stars a few thousand miles from earth rather than light years away. However, while not realistic in that respect, they are useful, in showing relationship of sextant altitude, Zenith Distance (ZD) and circles of equal altitude. A more accurate representation is provided on pages 37 and 38.

As the sextant altitude changes, so too does the distance (Zenith Distance, ZD) between the observer and the ground position of the celestial body change. As this Zenith Distance (ZD), (radius) changes, so too does the size of the circle of equal altitude surrounding the ground position of the celestial body change.

You will notice that the smaller the sextant angle, the greater the distance between the observer and the Ground Position (GP) of the celestial body.

For instance:

--A sextant angle of 0 degrees would place the observer 5,400 nautical miles (nm) distant from the Ground Position (GP) of the celestial body. (90 – 0) x 60 = 5,400)

A sextant angle of 90 degrees would place the observer directly beneath the celestial body, i.e. at the Ground Position (GP) of the celestial body. (90 – 90) x 60 = 0.)

•

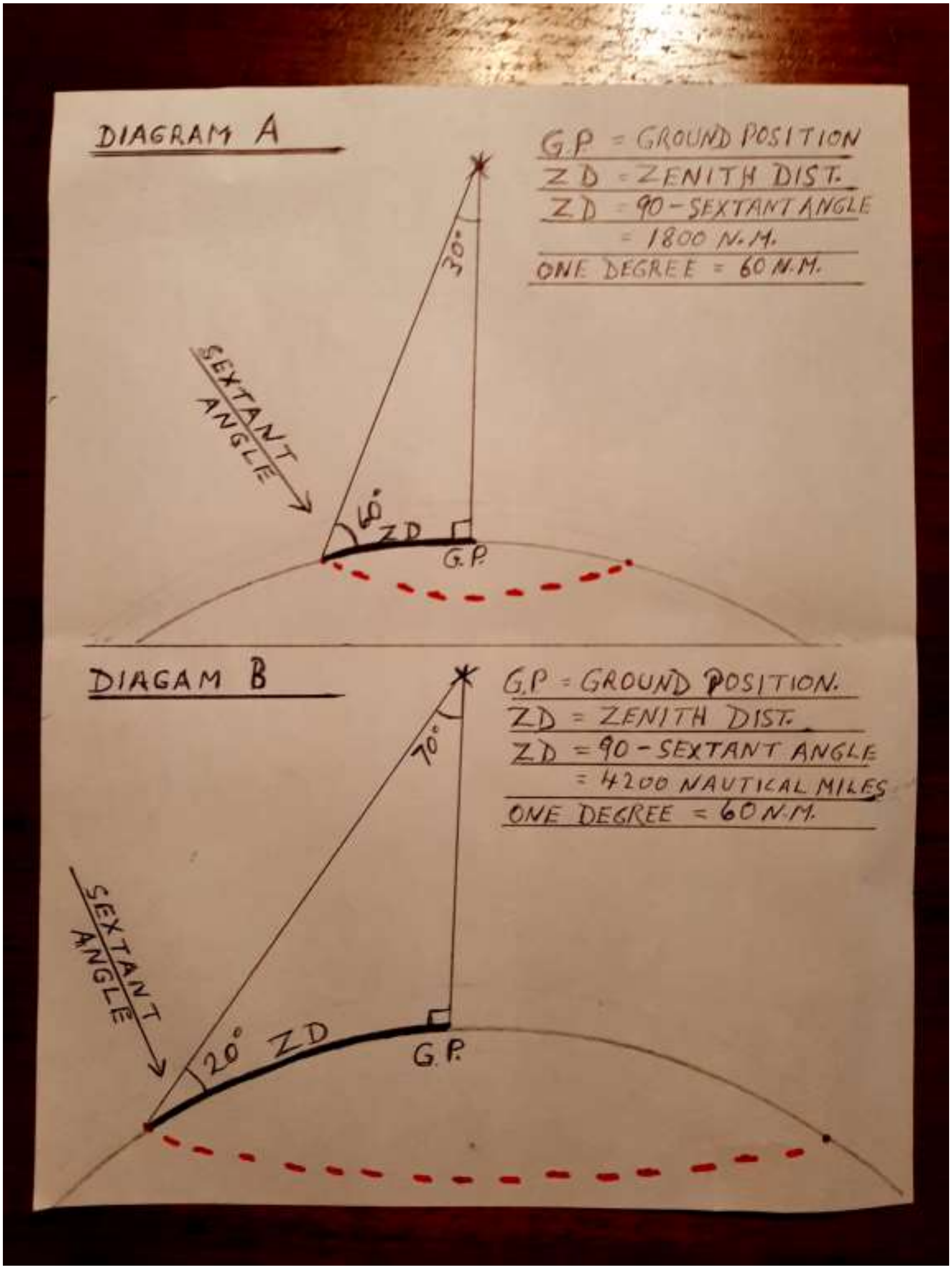


DIAGRAM A
G.P = GROUND POSITION
ZD = ZENITH DIST.
ZD = 90 - SEXTANT ANGLE
= 1800 N.M.
ONE DEGREE = 60 N.M.
30°
SEXTANT ANGLE
60°
ZD
G.P.
DIAGAM B
G.P = GROUND POSITION.
ZD = ZENITH DIST.
ZD = 90 - SEXTANT ANGLE
= 4200 NAUTICAL MILES
ONE DEGREE = 60 N.M.
70°
SEXTANT ANGLE
20°
ZD
G.P.

Diagram “A” above.
Sextant altitude (Ho) = 60 degrees.
Complimentary angle (30 degrees) of right angled triangle = Zenith Distance (ZD) angle.
Zenith Distance -- observer to Ground Position (G.P).
= 90 - 60 = 30 degrees = 1800 n.m.

Diagram “B” above.
Sextant altitude (Ho) = 20 degrees.
Complimentary angle (70 degrees) of right angled triangle = Zenith Distance (ZD) angle
Zenith Distance (observer to ground point (G.P)
= 90 – 20 = 70 degrees = 4200 n.m.
You will note that as the sextant altitude changes so does the radius -- Zenith Distance (ZD) -- of the circle of equal altitude.

The observer can be anywhere on these circles of equal altitude, (hatched in red in the above diagrams) for any particular sextant reading.

Imagine a street lamp to be a star. As you walk towards a street lamp the angle between your position on the pavement and the lamp increases as you get closer, until at 90 degrees the lamp is directly over your head, and the lateral distance is zero.

As you walk away the angle decreases. The farther away the smaller the angle.
Thus an angle equates to a distance.

The street lamp analogy, however, is valid only up to a point because:

(1) An object on the celestial sphere, such as a star, is visible far beyond our horizon and can have,-- and frequently does have,-- a ground position thousands of miles away. The stars, other than our Sun, are light years from earth. Distances of the sun and its orbiting planets are, while still vast, measured in light minutes. The moon’s average distance from the earth is around 216,000 nautical miles.

(2) The sun and the other stars are at such vast distances from us that their light is presumed to arrive at earth in parallel rays. (Diagrams on pages 37 and 39). Due to the curvature of the earth the horizon of the observer changes with reference to the horizon at the ground position of the star. Any star just “rising” or “setting” on the observer’s horizon would have a sextant altitude of 0 degrees. The ground position of such a star:-- (90 -0) x 60 -- would be 5,400 nautical miles from the observer. Any star directly above the observer’s head would have a sextant altitude of 90 degrees:-- (90 -90) x 60 = 0.

The observer would be at the ground position of the star.

(3) Also, unlike a street lamp, the earth is moving relative to the star, so the Ground Position (GP) of the star is

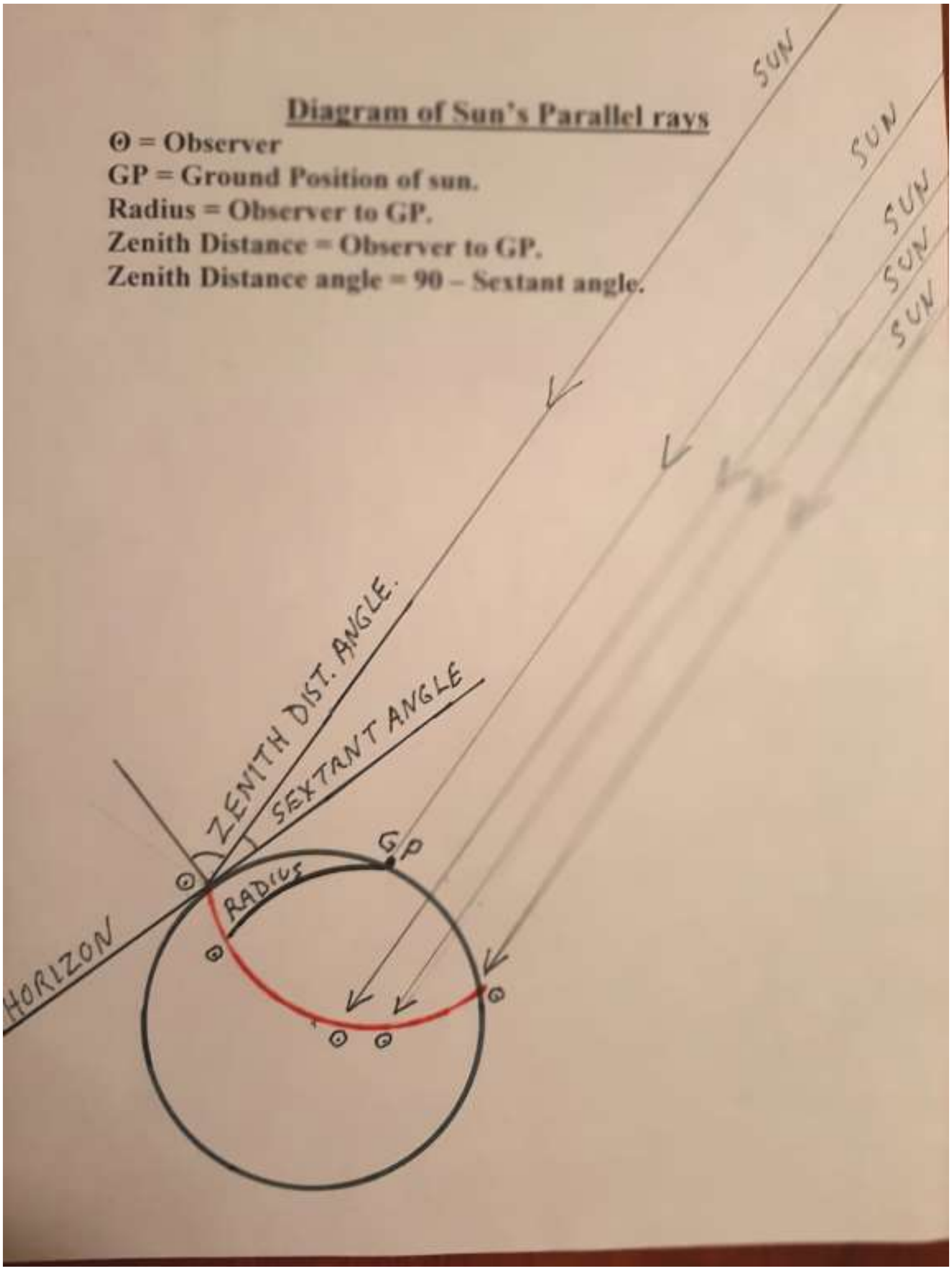
constantly moving, -- and its westward movement, due to the earth's rotation eastward, is rapid.

However the Nautical Almanac, entered with the date and the exact Greenwich Mean Time (GMT), gives us the star's Ground Position (GP) for that exact time, interpolated to the second. If, for example, your Greenwich Mean Time (GMT) is out by two seconds the Ground Position (GP) is out by one half of a nautical mile at the equator, i.e. an error of close to one kilometer.

The Nautical Almanac allows us to fix the star's position for any given second of time, by making it, as it were, fall to the ground at the time, (to the second), of the sight, and giving us the coordinates of that ground position (GP).

Ground Position (GP) is the position on the earth's surface directly beneath the celestial body. (Directly underneath the street lamp!)
The coordinates of the Ground Position (GP) of any celestial body are given in Declination (Latitude), and Greenwich Hour Angle (Longitude).
The lower the sextant reading the greater the distance, from the observer to the Ground Position (GP) of the observed body, the higher the sextant reading the less the distance. (Page 33). At a sextant reading of 90 degrees, the observer is directly beneath the celestial body, i.e. at the Ground Position (GP) of the celestial body.
Zenith Distance (ZD), --distance on the surface of the earth between observer and the ground position of the celestial body,-- can be expressed either as an angle in degrees, or as a distance in nautical miles. One degree =60 nautical miles.

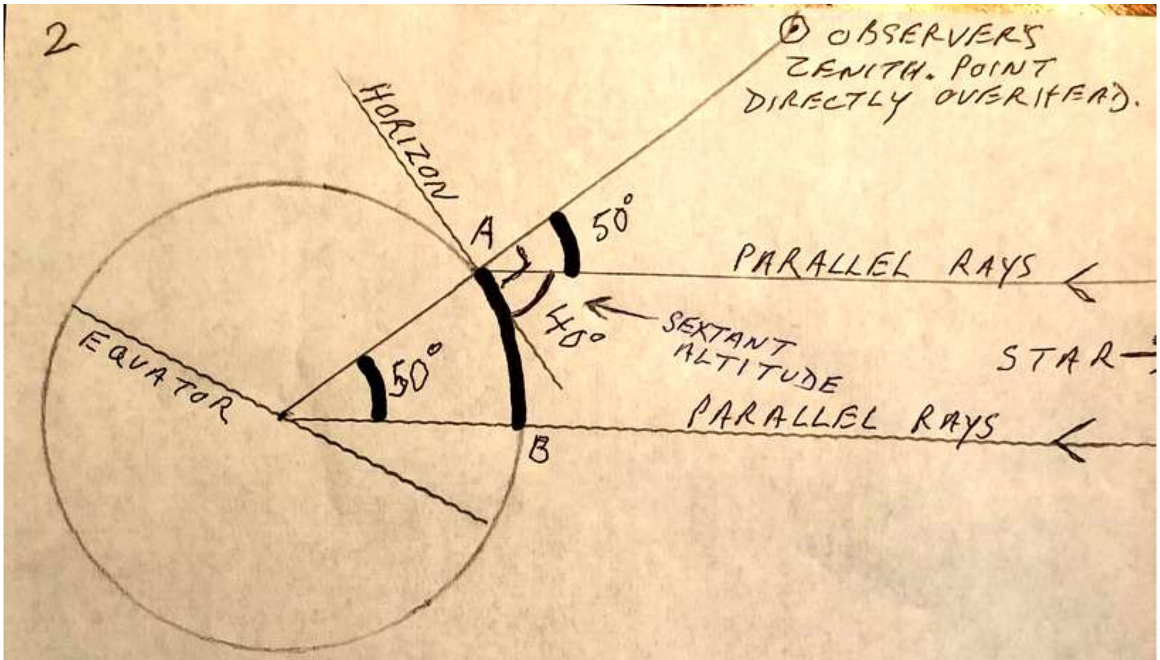
As an angle in degrees, Zenith Distance (ZD) is the complement of the sextant altitude (Ho)



The above diagram and the diagram on page 39 are more accurate representations of the principles of celestial navigation. The sun is so far from earth that its light is presumed to arrive at earth in parallel rays.
Any observer “o” anywhere on the circle of equal altitude (red) would get the same sextant reading, for the same celestial body at that same time.
The radius of the circle of equal altitude is observer to Ground Position (GP), -- is the Zenith Distance (ZD) in nautical miles.
Zenith Distance (ZD) is one of the basic principles of celestial navigation.

Below is the classic diagram used to explain Zenith Distance (ZD). Due to the Sun’s distance from earth its rays are depicted as striking earth in parallel rays.
The diagram explains how a sextant altitude relates to Zenith Distance (ZD), -- distance between observer and ground position (GP)

In the diagram below.
Sextant Altitude (Ho) = 40 degrees
Zenith Distance (ZD) = (90 - 40) = 50 degrees.
50 degrees = also angle at center of earth.
50 degrees x 60 = 3,000 n.m. on surface of earth.
Zenith Distance: Angle, and distance on earth in heavy ink.



Zenith Distance is a basic concept in Celestial Navigation

It is the distance between the observer and the ground position of the star,
It is A to B in the diagram and is expressed both as an angle at the center of the earth and as a distance in nautical miles.
Any angle subtended at the center of the earth represents a distance on the surface of the earth, One degree equaling 60 nautical miles.
In the diagram, A to B = 50 X 60 = 3000 nautical miles = Zenith distance.
Since the angle between the observer's zenith and the horizon is 90 degrees, and the sextant angle is 40 degrees..
90 minus 40 (sextant angle) equals 50, equals angle at center of earth in diagram, equals 3000 nautical miles on the earth's surface, equals Zenith Distance.
The star is so far away that its light can be represented as striking the earth in parallel rays.

In the above diagram "A" is the observer and "B" is the Ground Position (GP) of the star. The sextant angle is 40 degrees.

The star is directly above "B." "B" is the Ground Position (GP) of the star. The star's rays strike "B" at 90 degrees to the horizon at “B”. (If the earth were flat these parallel rays would strike “A,” the observer, 3,000 miles away, also at 90 degrees!)

But due to the curvature of the earth A has a different horizon to “B” and they strike “A” (the observer) at 40 degrees to that horizon, -- 40 degrees is the sextant angle.

Zenith Distance (ZD) = (90 - 40) = 50 degrees, which is the zenith distance angle, which multiplied by 60, = 3,000 nautical miles

The above is a plane view (a great circle cut through the center of the earth) but in effect “B”, as the Ground Position (GP) of the star, is the centre of a circle whose curved radius is the distance, (Zenith Distance) from “A” to “B.” Now visualize the above diagram as a three dimensional sphere. With “B” as the center, and curved distance “A” to “B” as the radius, swing a circle on the sphere of the earth, (Page 37). The observer “A” can be anywhere on the circumference of that circle whose radius (Zenith Distance) is “A” to “B”.

Note:

An interesting historical fact:

Eratosthenes, born 276 b.c., was a Greek mathematician who was also the librarian at the library of Alexandria. Using zenith distance he made an amazingly accurate estimate of the circumference of the earth.
He suspected, as did Pythagoras 250 years before him, that the earth was spherical in shape.

He had been told that at noon on the Summer Solstice the sun shone straight down into the bottom of a deep well at Cyrene, today's (Aswan) a city south of Alexandria. This would mean that the sun was directly overhead that place at that time. i.e. the well was the Ground Position (G.P.) of the sun.
At the same time in Alexandria, where he lived, he was aware that the sun cast a shadow on that same day, and thus could not be directly overhead. He reasoned that the earth was a sphere, and set out to measure its circumference. He measured the distance of the shadow cast by a tower of known height at Alexandria where he lived.
Eratosthanes did not have a sextant but knowing the height of the tower and the length of the shadow cast by the sun he was able to ascertain the other two complimentary angles of this right angled triangle,
-- 82.5 degrees and 7.5 degrees.
82.5 degrees, being the angle from the tip of the shadow to the top of the tower would be equivalent to sextant altitude. Thus the complimentary angle 90 – 82.5 = 7.5 would be the Zenith Distance (ZD) angle.
The distance between Alexandria and Cyrene was already know to be 5,000 stadia, (a distance equivalent to about 434 nautical miles). The "stadia", a Greek measurement of distance, was approximately 600 feet.

Let's first use the above diagram, and zenith distance to measure the earth's circumference:
In the diagram: "A" can represent Eratosthenes at Alexandria and "B" can represent the sun's Ground Position (GP). at Cyrene.

In the above diagram:
If 50 degrees = 3000 n.m.
Then 360 degrees, (earth's circumference) = 360 x 3000 divided by 50 = ***21,600 n.m***. -- which is earth's actual circumference.
Using Eratosthenes' figures and converting his "stadia" distance to nautical miles his equation would be:
If 7.5 degrees = 434 nautical miles, then
360 degrees (earth's circumference) = 360 x 434 divided by 7.5 = ***20,832 n.m***.--
-- an error of less than 1%!

Was Christopher Columbus unaware of Eratosthenes' calculations, when 1700 years later he underestimated the circumference of the earth by 5,300 nautical miles, -- an error of 25%!

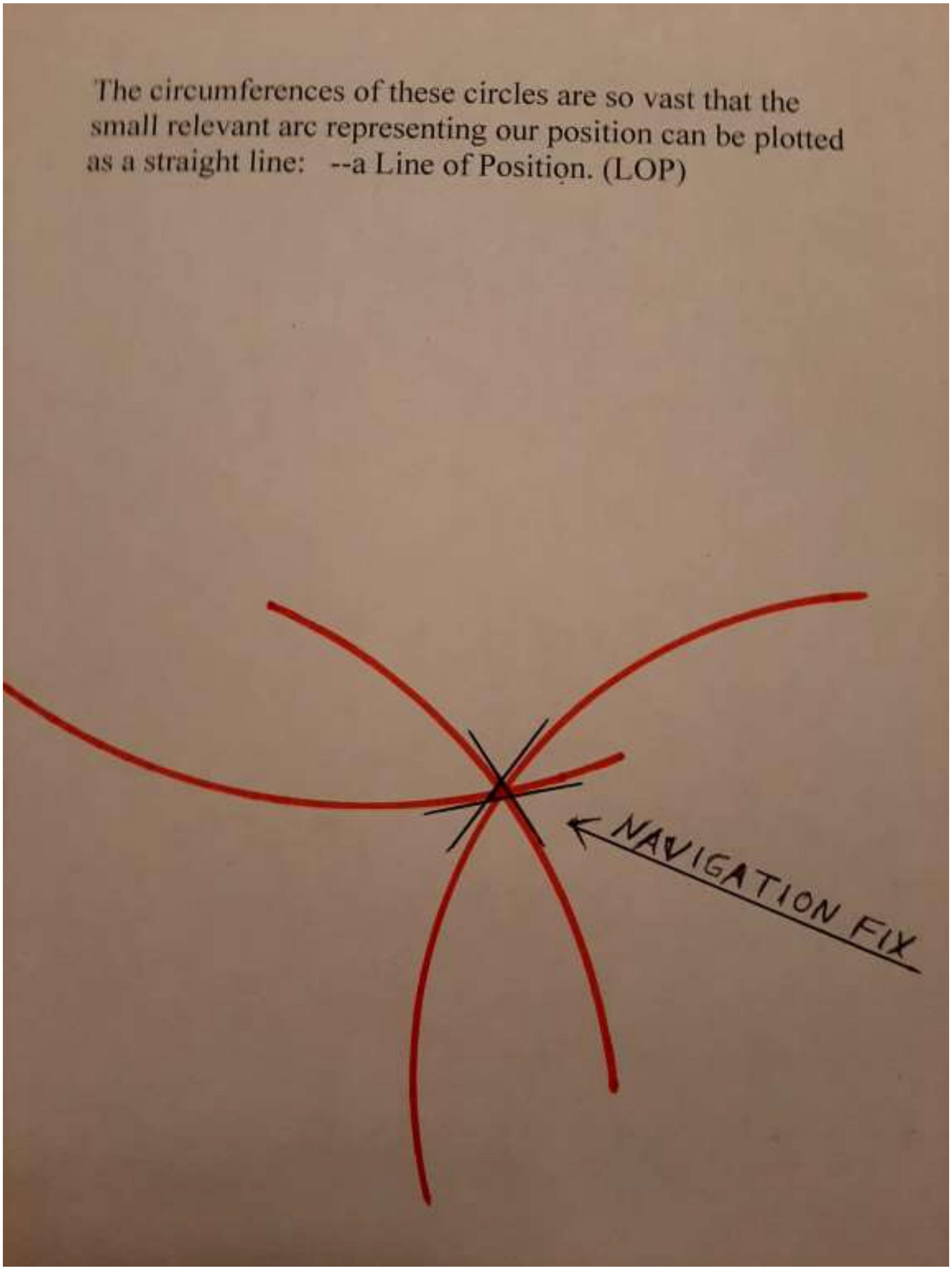
Again the principle.

--_Measure with a sextant the angular height of a celestial body above the horizon.

--_Enter the Nautical Almanac with the date and exact Greenwich Mean Time (GMT) of the observation to find the Ground Position (GP) of the observed body for that time (to the second). This Ground Position (GP) is given in Greenwich Hour Angle (GHA) -- (Longitude), and Declination -- (Latitude)

The observer is somewhere on that circle of equal altitude around that ground position, that equates to the sextant altitude. (Diagrams on Pages 3,4,31, 33, 37.) Take sights on two more bodies, (Pages 3 and 4), and the observer is somewhere on the circles of equal altitude surrounding each of these bodies also.
The only place that is common to all three circles, is where all three circles intersect. This is were the observer is.

Diagram of intersecting circles



If we take, as an example, a sextant altitude of 20 degrees, the circle of equal altitude would be a circle with a radius of 4200 nautical miles. (90-20) x 60= Zenith Distance = distance observer to ground position = 4200 nautical miles).

(Accuracy of sextant altitudes below 20 degrees are problematic due to increasing, and more variable, atmospheric refraction at lower sextant altitudes).

Thus a specific angle corresponds to a specific distance, and to a specific sized circle of equal altitude around the Ground Position (GP) (Page 33). In celestial navigation it is easier to deal with the angles, rather than the distances.

Let's say that a mariner, somewhere in the South Atlantic sights a sextant altitude of 20 degrees on a celestial body. Consulting the nautical almanac (and a map of the world!) he finds that the observed body for that time has a Ground Position (GP) in the Atlantic Ocean between Africa and Brazil, 4,200 nautical miles away. This would place the observer *somewhere* on a circle of equal altitude with a radius (Zenith Distance (ZD) of 4,200 nautical miles, -- and a circumference passing through:

Tierra del Fuego, the Pacific Ocean, Mexico, Michigan, Hudsons Bay, Greenland, Norway, The Black Sea, the Arabian Peninsula, Madagascar, the Indian Ocean and the South Atlantic Ocean. (Diagram Page 4.)

You will see from this, that the circle of equal altitude around the ground position of a celestial body can extend over many thousands of miles, passing through different hemispheres, different continents, and different oceans.

It does however give the observer the information that he is *somewhere* on that vast circle, and by taking sights on at least one, preferably two, more celestial bodies the mariner can definitively fix his position.

While the principle is simple, the problem in Celestial navigation arises when we attempt to plot these circles of such huge circumferences on a chart. How do we plot a Ground Position (GP) that is not on our chart and that is 4,200 miles distant from us? In the above example of a sextant altitude of 20 degrees we would need a chart that covered nearly half the globe!
We could perhaps draw these circles on a globe, but at that scale the thickness of the pencil line alone could equate to something like 40 nautical miles.

High sextant altitudes, above 85 degrees, (which would place the observer closer to the ground position) are also problematic, due to technical issues.

In practice, the optimal sextant altitude range is considered to be any altitude between 15 and 65 degrees.

It was not until as late as 1875 that a French admiral with the unassuming name of Adolphe-Laurent-Anatole Marque de Blonde de Saint-Hilaire came up with a method, known as the **"*Intercept Method*"** which is now the method commonly used in celestial navigation to plot lines of position on a chart.

As you become increasingly familiar with this method, and as you learn to appreciate its elegance, you will no doubt be tempted to throw your chapeau in the air with Gallic abandon, and shout: "***VIVE LA FRANCE***"

However it is worth mentioning that in developing this Intercept method, St Hilaire had stood on the shoulders of another mariner; an American merchant marine captain named Thomas Sumner. Sumner, close to 40 years earlier, had developed the concept of using the circle of equal altitude to plot what was known as a “Sumner Line”.

Sumner who had entered Harvard at age 15, subsequently enlisted as a common sailor in the merchant marine. He rose to be a captain, but sadly, his once keen intellect destroyed, he ended his days in an insane asylum.

The object is to find a mechanism by which one can plot on our chart, as a line of position (LOP), that small arc of the huge circle of equal altitude that corresponds to our observed altitude, (Ho).

Part 2. Plotting a Line of Position

Since this Intercept Method is so integral to the plotting of a Line of Position (LOP) on our chart it will be dealt with in some detail, (and with some repetition!) below.

Essentially what this “Intercept Method” does is to allow us to superimpose, on our chart, that small relevant part of the often vast circle of equal altitude that corresponds to our sextant altitude (Ho). i.e. that part of the circle that passes south of the Cape of Good Hope (Page 4) in the region of our dead reckoned position, rather than that part of the circle that passes through Tierra del Fuego, Michigan or Greenland!

That small arc of the circle, plotted as a straight line, becomes our first line of position (LOP).

How we do this is to use our Dead Reckoned (DR) position (or an assumed position close to it), as a reference point, from which we can plot our actual circle of equal altitude (Ho) on our chart.

We know by our dead reckoning that we are somewhere on our chart, (hopefully!)

We now pick a position on our chart close to our dead reckoned position. This we call an *assumed*, -- or more appropriately a *reference* position.

We can use our Dead Reckoned (DR) position as the Assumed Position (AP), (Page 72), but by selectively choosing an Assumed Position (AP) somewhere close to it, but using whole degrees of latitude and whole degrees of hour angle, we can make subsequent calculations easier.

- (see Pages 81 and 83)

From this assumed, or reference, position we compute mathematically the altitude we would obtain, were we to take a sighting from this Assumed Position (AP) on the same body and for the same time as sighted for out prior observed sextant altitude (Ho). (It is here that the math involved in solving spherical triangles enters the picture, (Page 78). This computed altitude, now referred to as (Hc), is the altitude we would get if we had actually taken the sight, on the same body and for the same time, from this assumed position on our chart.

The math, and the inputs used, to compute this altitude from the assumed, or reference, position is explained commencing on page 76 in the text in red.

The prior observed sextant altitude (Ho), and this now computed altitude (Hc), relate to each other in that they both share the same celestial body, for the same time,-- and therefore the same Ground Position (GP).

Since both of these altitudes, observed (Ho) and computed (Hc), have the same Ground Position (GP) we are now dealing with two concentric circles, the radius (Zenith Distance), of each of which we know,-- one circle for our computed altitude (Hc), and one for our observed sextant altitude (Ho). We know that the circle of equal altitude for one of these concentric circles,-- the one for our computed altitude (Hc),-- passes through our Assumed Position (AP) on our chart. Where in reference to this assumed position (AP) does our circle of equal altitude for our actual observed sextant altitude (Ho) lie?

Depending on whether our observed sextant angle (Ho) is greater or is lesser than our computed angle (Hc) the circumference of the circle of equal altitude for Ho will fall either inside or outside the circumference of the circle of equal altitude for our computed altitude (Hc).

If Ho is the lesser angle it will fall outside. If Ho is the greater angle it will fall inside.

In Diagram A on Page 50 below, Ho, falls outside,-- known as Intercept Away.

In Diagram B on page 51 below, Ho, falls inside ,-- known as Intercept Towards.

Moreover, we can now also plot the exact distance, -- inside or outside. This distance will be determined by the amount of difference in radii between these two concentric circles.

By comparing the difference in radii, Zenith Distances (ZD), between these two concentric circles, we can now plot on our chart, as a Line of Position (LOP), exactly where our circle of equal altitude, for our actual sextant altitude (Ho) lies in relation to our assumed position (AP)

(Diagrams A and B below, Page 50)

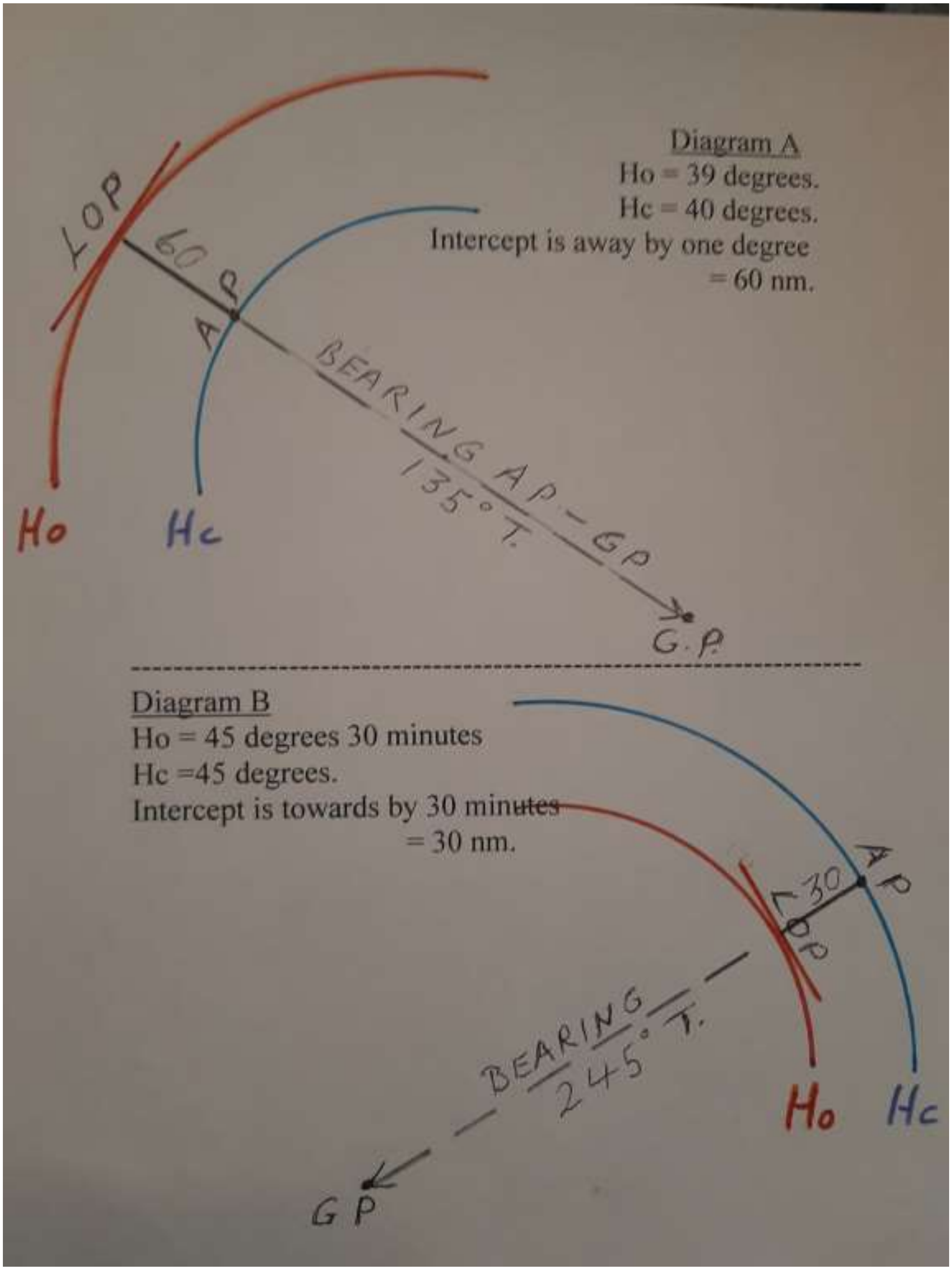
--[When we mathematically compute an altitude at the Assumed Position (AP), in addition to a computed altitude (Hc) we also obtain another very useful piece of information, an azimuth angle from which we derive a True Bearing from the Assumed Position (AP) to the Ground Position (GP)]-

Let us use some figures to illustrate how we can now; by comparing our observed sextant altitude (Ho) with our computed altitude (Hc), and using our Assumed Position (AP) as a reference, draw a **Line of Position (LOP)** on our chart. (Diagram A).

Observed altitude (Ho) = 39 degrees,

Computed altitude (Hc) =40 degrees.

Bearing from assumed position to distant ground position is 135 degrees True (135T)



Through our Assumed Position (AP) on our chart we extend a bearing line of 135 degrees True. (Diagram A)

Since, in this example, our observed sextant altitude (Ho) is the smaller angle, (larger radius) our circle of equal altitude for Ho must fall outside the circle for our computed altitude (Hc), and it is outside by the difference between Ho and Hc, which in this example is one degree, which is 60 nautical miles.

From our Assumed Position (AP) we mark off along our bearing line 60 nautical miles in a direction farther away from the distant ground position than is our reference AP, and there draw a line at right angles to this bearing line. This line a, tangent to the circle for Ho, now represents that small relevant part of the circle of equal altitude for our observed sextant altitude (Ho), and is our first **Line of Position (LOP).** In order to obtain a navigational fix, at least one more line of position, intersecting this first, would be needed.

The difference, converted to nautical miles, between observed sextant altitude (Ho) and computed altitude (Hc) is known as the **Intercept.** When Ho is the greater angle the Intercept is **towards** i.e. closer to the Ground Position (GP) than is the Assumed Position (AP). When Ho is the lesser angle the intercept is **away,** i.e. farther from the Ground Position (GP) than is the Assumed Position (AP) -- (Diagram A, Page 50). Had our observed sextant altitude (Ho) been the greater angle the intercept would have been **towards**.-- (Diagram B, Page 51)

The above diagrams (not to scale), show the two possible scenarios in plotting a Line of Position (LOP) from our Assumed Position (AP), by comparing Ho to Hc. The larger the angle, (observed or computed) the smaller the circle, and the closer to Ground Position. (GP). (Page 33).

In diagram A above, Ho is the lesser angle by one degree, and therefore the intercept is away by 60 nautical miles.

In diagram B, Ho is the greater angle by 30 minutes of arc, and therefore the intercept is towards by 30 nautical miles.

…………………………………………………………..

The diagram A1 below is an elevation view, used as one further demonstration of the Intercept Method for plotting a Line of Position. In this diagram the Intercept is Away by one degree.

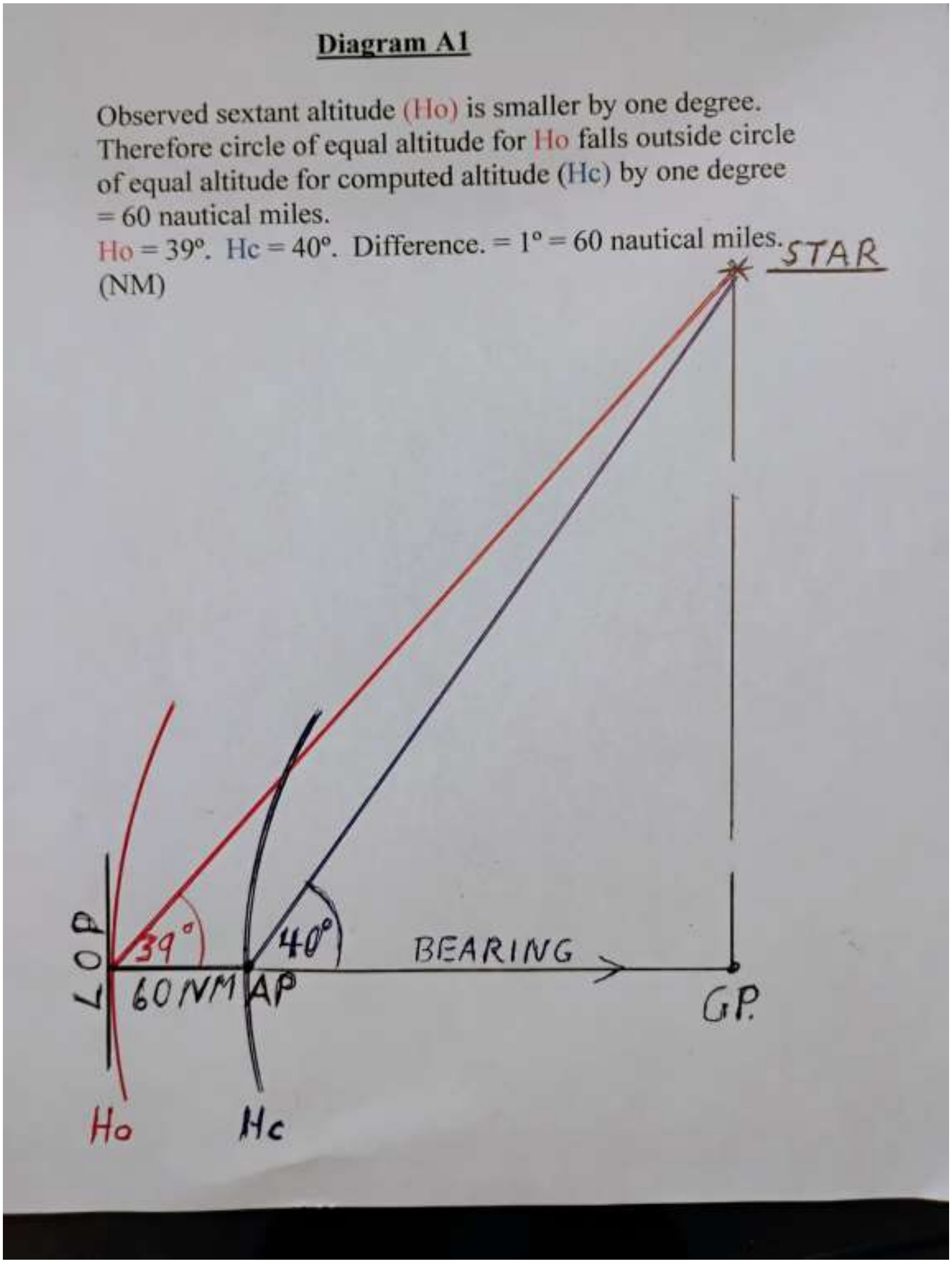


Diagram A1

Observed sextant altitude (Ho) is smaller by one degree. Therefore circle of equal altitude for Ho falls outside circle of equal altitude for computed altitude (Hc) by one degree = 60 nautical miles.

Ho = 39º. Hc = 40º. Difference. = 1º = 60 nautical miles. (NM)

<u>Note:</u> *Let us here take a moment to marvel at the genius of Marc St. Hilaire, who had the inspiration to pick a location on his chart where he was not in order to find that location on his chart where he was! Thanks to this man's genius we have now narrowed down our position, from somewhere on a vast circle that encompassed half the globe, to somewhere on a line on our chart a few miles long. Another line intersecting this first and we have fixed our position.*

<u>Note</u>: *Some of these, not to scale diagrams, will show an intercept in the order of one whole degree, (i.e.60 nautical miles).This is for clarity of demonstration only. In practice, the Intercept is usually a matter of minutes of arc, rather than degrees. If confronted with such a large intercept, in the order of one whole degree, one might question the accuracy of one's calculations*

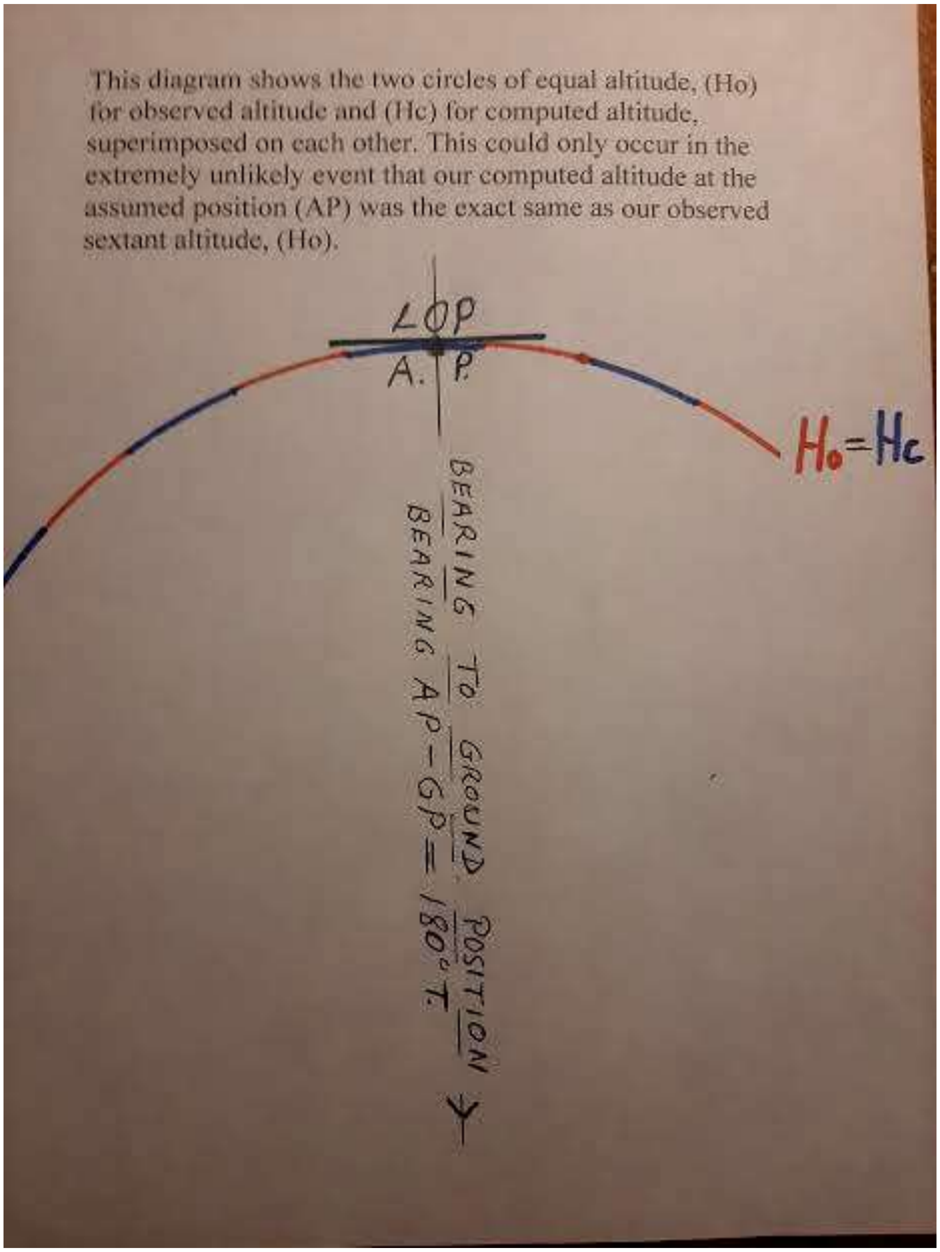
As a further means of explanation, let's say that, by the merest chance, (in a rather unlikely third scenario!), this computed altitude (Hc) taken at our Assumed Position (AP) turned out to be the very same altitude as our observed sextant altitude (Ho), (Diagram Page 56). This would mean that totally by accident, the Assumed Position (AP) we had chosen on our chart lay on the circle of equal altitude for our actual observed sextant altitude (Ho). The two circles of equal altitude, the one observed (Ho) in red (below), and the other computed (Hc) in blue (below), would have the same radius, (zenith distance) and would be superimposed on each other. It would mean that the intercept was 0, (neither <u>away</u> nor <u>towards)</u>. Our line of position (LOP) for Ho, in this unlikely scenario, would pass through our assumed position (AP), rather than being removed from it, by the amount of the Intercept; (Diagram below)

▼? Question:

Would our assumed position (AP) then, in this scenario, be our actual position?

Please try to reason out the answer for yourself, before looking up the answer at end of tutorial on page 126.

Reasoning out the answer will help with a better understanding of the intercept method, used in plotting a line of position (LOP).



In practically every instance however, Ho will differ from Hc, and they will differ from each other by the differences in the radii, (zenith distances), of these two concentric circles. These two circles will have differing radii,-- zenith distances, (Diagrams on pages, 51 and 62), the one for our observed sextant altitude (Ho) and the other for our computed altitude (Hc). But we know the radius in nautical miles of each circle, -- zenith distances-- and we now also have a reference point on our chart, -- our assumed position (AP), -- from which we can plot this *difference* in radii of these two concentric circles. This difference is the **Intercept**.

We are now able to plot our actual line of position (LOP) on our chart, -- i.e. that part of the circle of equal altitude corresponding to Ho. The difference in radii of the two circles, -- the difference, converted to nautical miles, between observed sextant altitude (Ho) and computed altitude (Hc) -- is measured along the bearing line. This line, which passes through our Assumed Position (AP) is a true bearing from our assumed position, (AP) to the distant Ground Position (GP).

Despite the fact that the circle of equal altitude for Ho is so huge, we have found a mechanism to plot, *on our chart,* the small relevant part of the circumference of that circle that relates to our position.

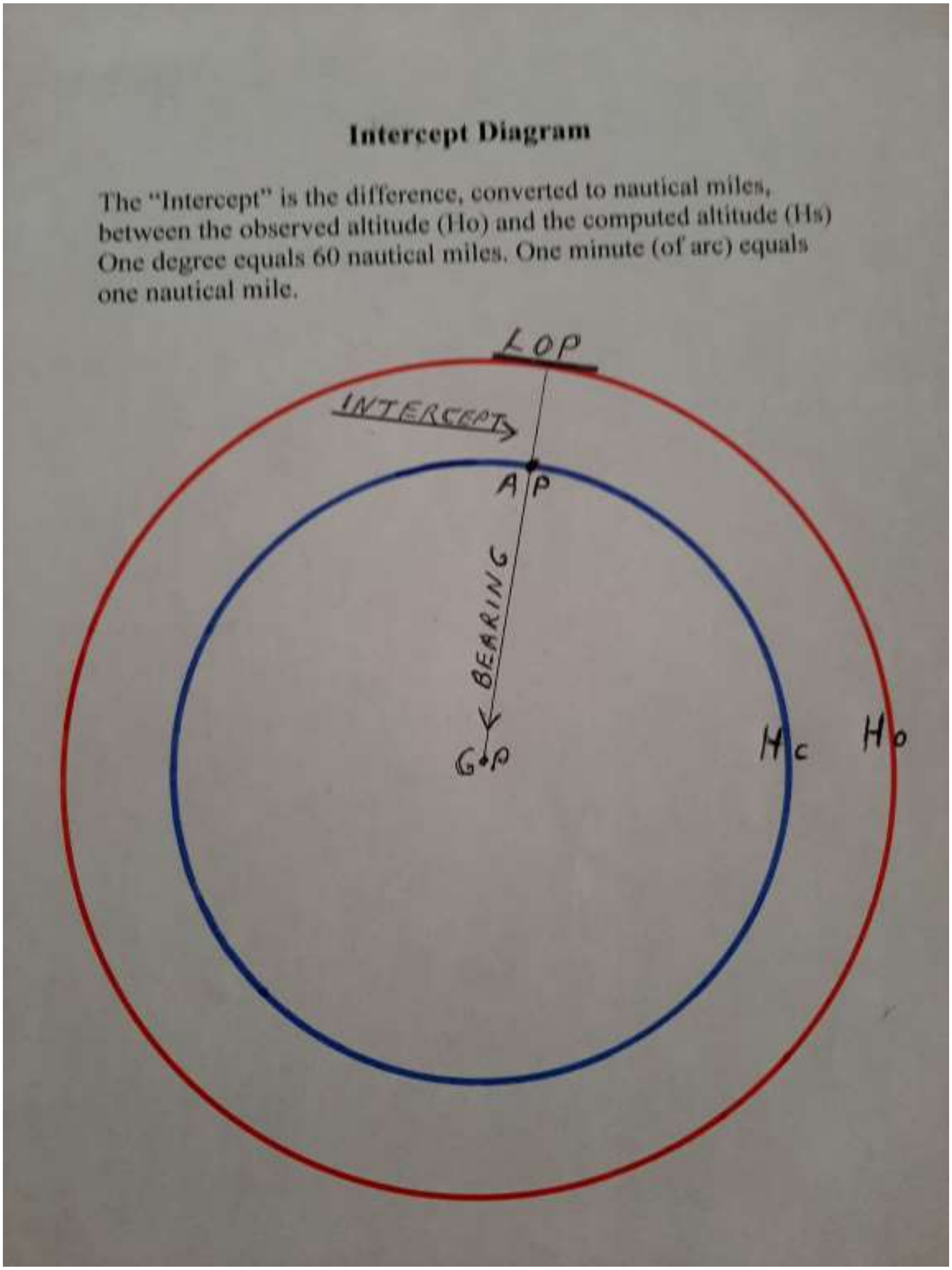
Plotted as a straight line it is our first Line of Position (LOP).

This difference in the radii of the two circles is also the difference in their zenith distances, -- expressed as either an angle or a distance. It is also the difference between observed sextant altitude (Ho), and computed altitude (Hc). It is referred to as the **Intercept**. Remember the larger the angle, (observed or computed) the smaller the circle of equal altitude, and the closer to the ground position. (GP), -- and vice versa.

The diagram below on page 60 shows how, in general, a line of position is plotted using this Intercept Method. Subsequent diagrams, using specific values for observed sextant altitude (Ho), and computed altitude (Hc), and true bearing of Assumed Position (AP) to Ground Position (GP), will provide more specific examples and explanations of the concept.

The Assumed Position (AP) on our chart and the bearing line passing through it provides us with a reference from which we can plot our actual Line of Position (LOP).

Our actual Line of Position (LOP) is that small arc of the circle of equal altitude relating to our actual observed sextant altitude (Ho)



Intercept Diagram

The "Intercept" is the difference, converted to nautical miles, between the observed altitude (Ho) and the computed altitude (Hs) One degree equals 60 nautical miles. One minute (of arc) equals one nautical mile.

In above diagram: Intercept is <u>Away.</u>

(Error) In the text in this diagram above, "Hs" should read "Hc," which is computed altitude.

In plotting the intercept we want to know where is our circle of equal altitude for our observed sextant altitude (Ho) in relation to our circle of equal altitude for our computed altitude (Hc). We know that the circle of equal altitude for our computed altitude (Hc) passes through our Assumed Position (AP).

Where is the circle for Ho in relation to this assumed position (AP)?

Our line of position (LOP) will lie on the circle for Ho, and is represented as a straight line at right angles to the extended line of bearing from Assumed Position (AP) to Ground Position (GP).

We want to know;

(1) Is our actual observed circle of equal altitude for (Ho) closer in towards that distant Ground Position (GP) ***--"Intercept towards"--*** or is it farther out from the Ground Position (GP) -- ***"Intercept away"*** **--** than is our Assumed Position (AP)?

(2) and we want to know, by ***how much?***

--By comparing our actual observed sextant altitude (Ho) to our computed altitude (Hc) we can answer both of the above questions.

By comparing Ho to Hc we are also comparing the radii,**--** the Zenith Distances,-- of two concentric circles.

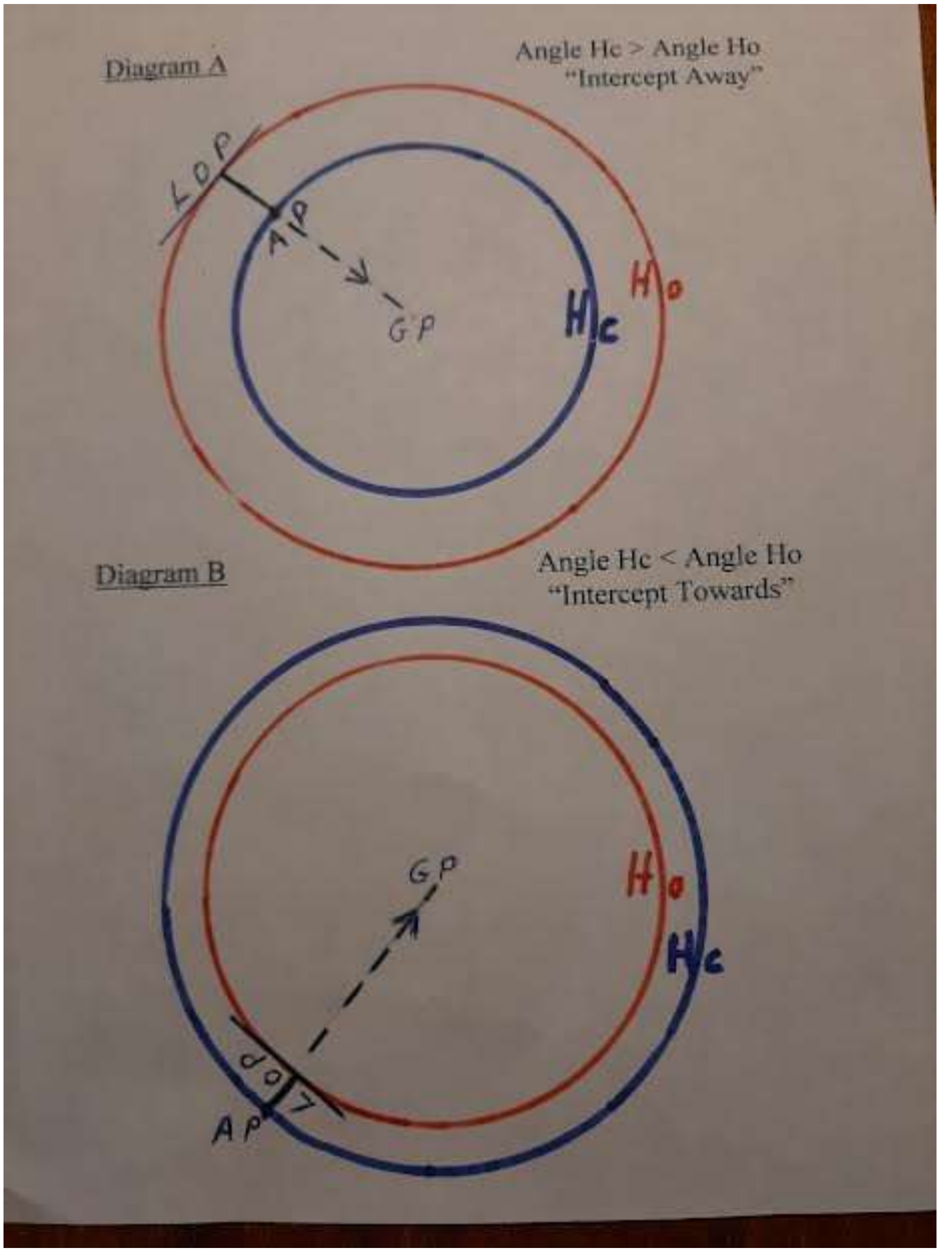
It will be away or towards by the amount of the difference in degrees, converted to nautical miles (n.m.), between Ho and Hc. One degree = 60 nautical miles (n.m). One minute (of arc) = One nautical mile (name.).

One way to remember whether the intercept is “towards” or “away” is to think of an imaginary Japanese admiral: --

“Homoto”

(Ho) more intercept towards.

As a further example let’s give values to the two scenarios below:



<u>Diagram A above</u>:

Computed altitude   Hc = 40 degrees. (Blue Circle).

Observed altitude   Ho = 39 degrees.(Red Circle).

Bearing of Assumed Position (AP) to Ground Position (GP) 135 degrees true. (135T) Since observed angle Ho is less than computed angle Hc by one degree, the intercept would be 60 nautical miles “away”

Diagram B above:

Observed altitude, Ho = 20 degrees 30 minutes.(Red Circle)

Computed altitude, Hc = 20 degrees 05 minutes.(Blue Circle)

Bearing of assumed position (AP) to ground position (GP)

is 045 degrees true, (045T)

Since observed angle Ho is greater than the computed angle Hc by 25 minutes of arc, the intercept would be 25 nautical miles “towards”

(HOMOTO)

In Diagram A above, the observed sextant altitude (Ho), shown as a red circle, has the larger radius (ZD) and must thus be the smaller angle and farther from the Ground Position (GP). The smaller the angle (observed or computed), the larger the radius (zenith distance) and the farther from the ground position (GP).

In Diagram B the observed sextant altitude (Ho), shown as a red circle, has the smaller radius and must thus be the larger angle and closer to the Ground Position (GP). The larger the angle, (observed or computed) the smaller the radius, (zenith distance) and the closer to the Ground Position (GP).

In both diagrams these two circles are separated from each other by:

-- The differences between Ho and Hc.

-- The difference in their zenith distances (ZD).

-- The differences in their radii..

-- The Intercept.

We know this difference and we also have a position on our chart, our Assumed Position (AP), from which we can plot this difference.

This difference, -- **the Intercept**,-- will be the distance on our chart that separates the circumferences of these two concentric circles. Thus we now can plot on our chart, as a **Line of Position (LOP),** that relative part of the circle of equal altitude that corresponds to our **observed sextant altitude (Ho).**

We have now found a mechanism by which we can plot, as a line of position (LOP), that relevant portion of our observed circle of equal altitude on our chart:

We have now plotted **one** *Line of Position (LOP)*

To obtain a Navigational Fix we need to perform this same procedure on at least one more celestial body,-- preferably two more, -- but we need at least one more intersecting circle.

If another celestial body is not available we can sight the same body again after the passage of a suitable time. Such as; Sun in the morning and Sun in the afternoon.-- A running fix. (Page 92.) The line of position (LOP) obtained for the morning sight is advanced along the vessel's dead reckoned track to obtain a two position line navigational fix.

Commencing Text in Red

To plot a line of position (LOP) by the intercept method we need to know:

--Observed sextant altitude (Ho)

--Computed altitude (Hc)

-- The true bearing from assumed position (AP) to Ground Position (GP)

Let's again assume values for the above three quantities in order to plot a line of position: (Diagram Page 68)

Computed Altitude (Hc) = 50 degrees.

Observed sextant Altitude (Ho) = 49 degrees.

Bearing AP to GP = 240 degrees true (240T)

The Intercept, which is the difference between Hc and Ho, is one degree, which equals 60 nautical miles.

We draw a true bearing of 240 degrees extended through our assumed position (AP).

Since, in the above example, our observed sextant altitude (Ho) is *smaller* than our computed altitude (Hc) by one degree (60 nautical miles) we measure off 60 nautical miles along this extended bearing line *away* from the Assumed Position,

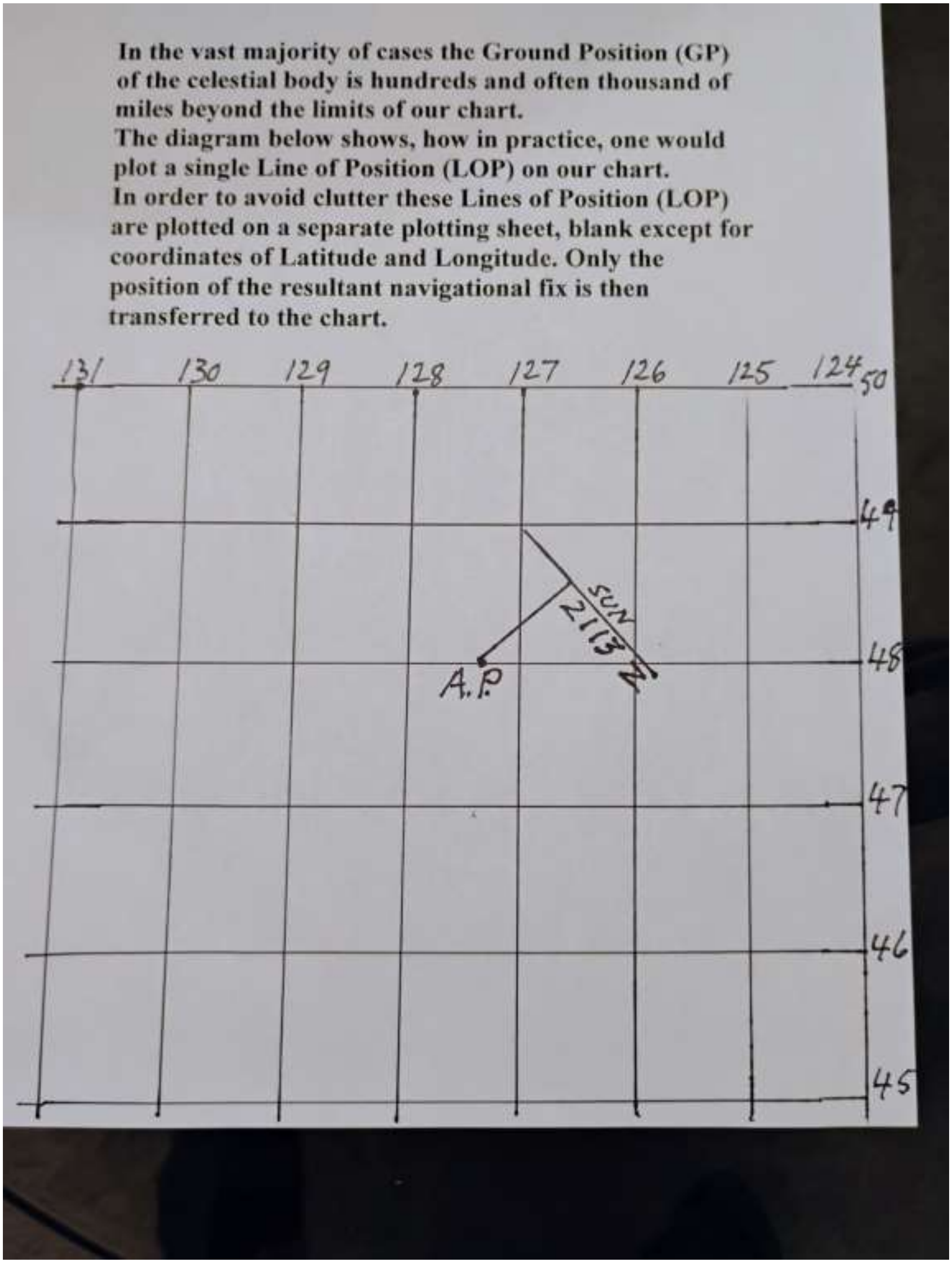
--"*Intercept away.*" **--** (This distance is measured with dividers using the latitude scale on the side of the chart).

At this point we draw a line at right angles to our (extended) true bearing line to represent that part of the circle of equal altitude corresponding to our observed altitude (Ho), and this becomes **our Line of Position (LOP).**

We have now obtained one Line Of Position (LOP) on our chart. In order to obtain a fix we need to perform the same exercise again with at least one other observed sextant altitude (Ho), on either the same body after a sufficient time lapse, (a running fix -- page 92), or on another body.

By converting the difference in altitudes between Ho and Hc to nautical miles and using our Assumed Position as a reference we can plot the relevant part of the circle of equal altitude for our observed altitude (Ho) on our chart.

So far in these not-to-scale diagrams the true bearing line is shown plotted all the way to the distant Ground Position This is for explanatory purpose only, and would not be done, -- nor could it be done, -- in actual plotting. As you will see in the diagram below, only that part of the bearing line corresponding to the intercept need be plotted.

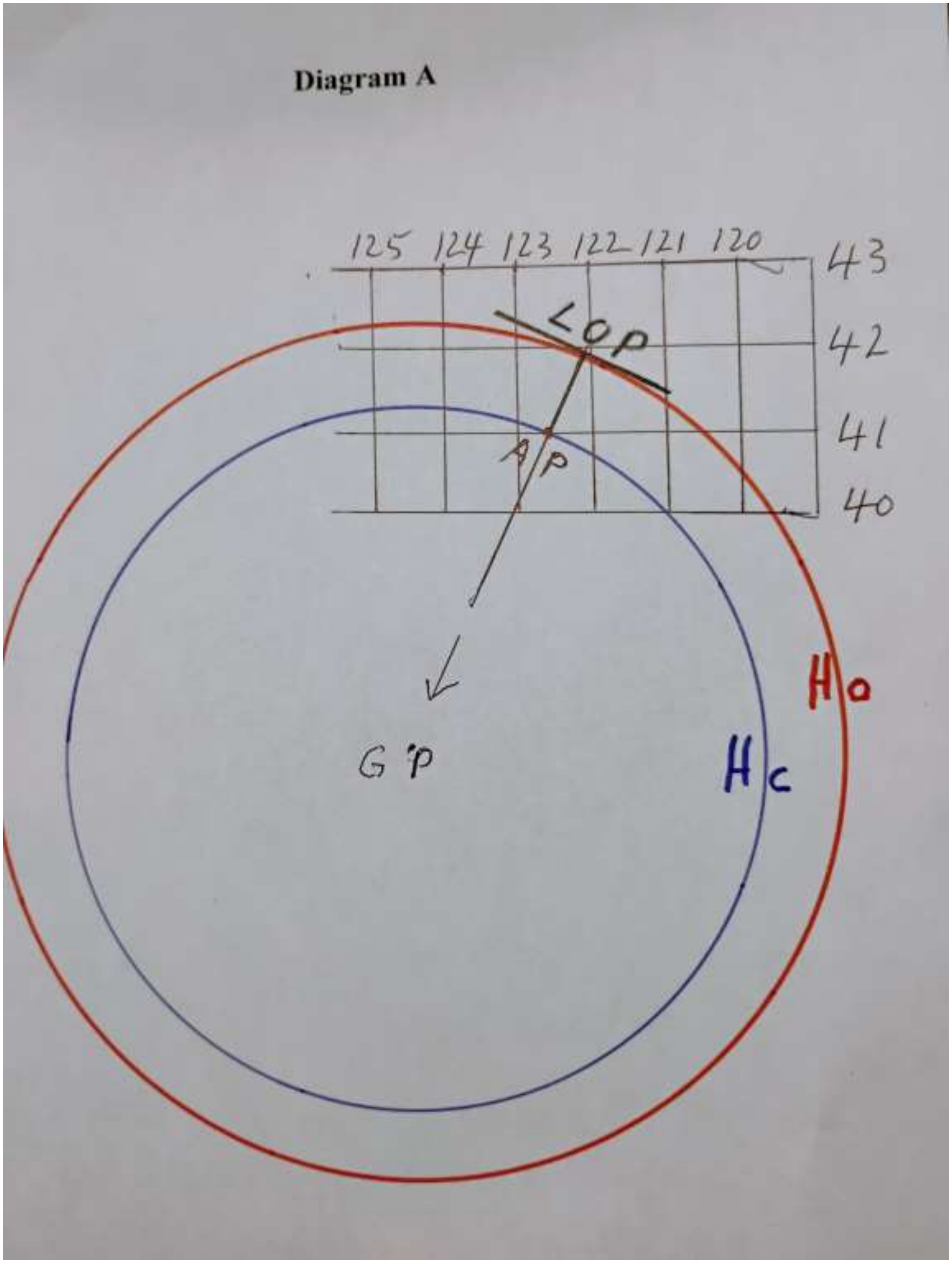


In the vast majority of cases the Ground Position (GP) of the celestial body is hundreds and often thousand of miles beyond the limits of our chart.
The diagram below shows, how in practice, one would plot a single Line of Position (LOP) on our chart.
In order to avoid clutter these Lines of Position (LOP) are plotted on a separate plotting sheet, blank except for coordinates of Latitude and Longitude. Only the position of the resultant navigational fix is then transferred to the chart.

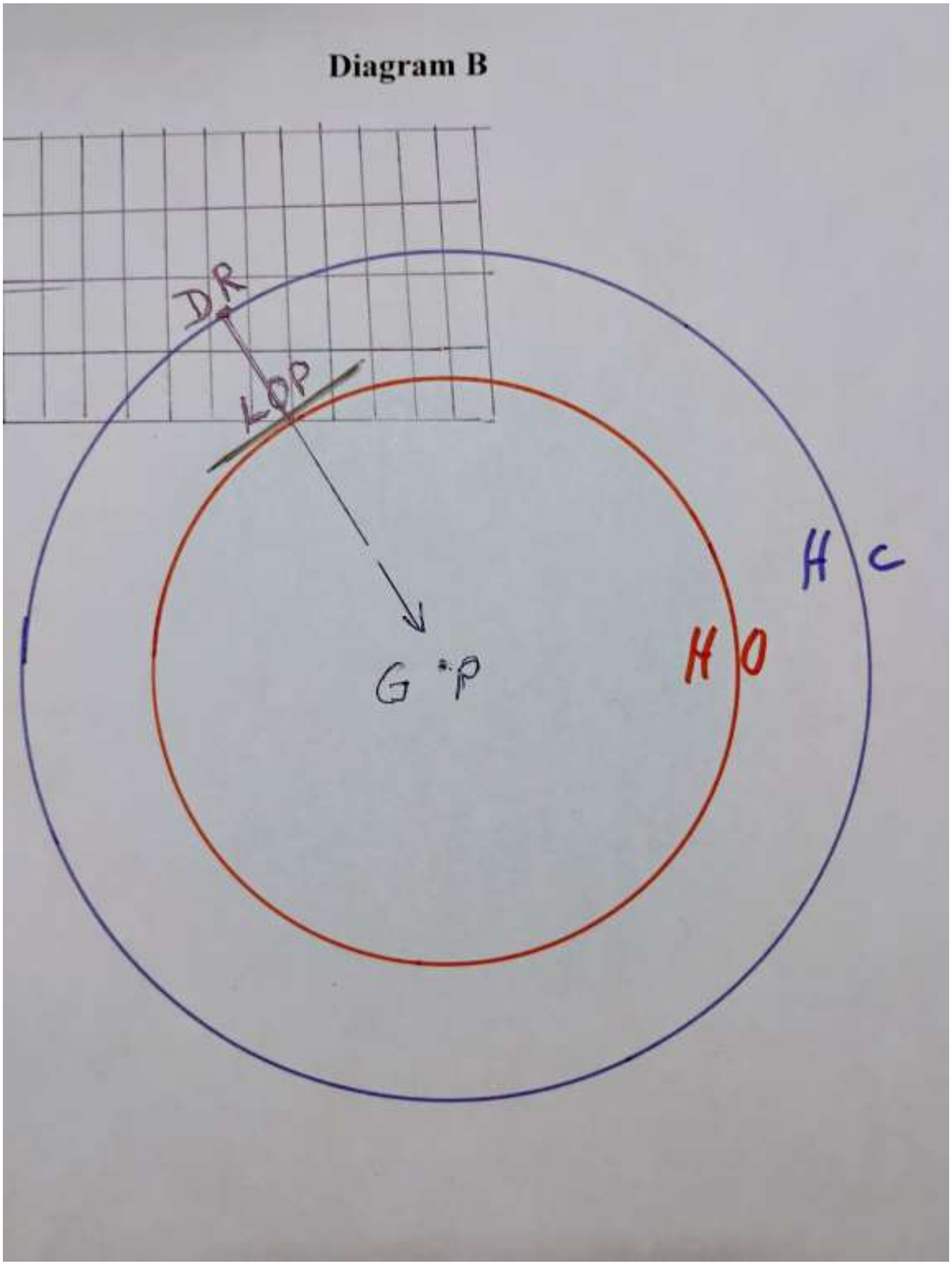
The True Bearing line is extended through the AP.

The Intercept, 60 nautical miles in the above example, is measured from latitude scale by side of chart. One degree Latitude = 60 nautical miles.

Some more examples of plotting a Line of Position (LOP). In Diagram A, below, true bearing AP to GP = 210 degrees. (210T)



In Diagram A, above, the observed sextant altitude (Ho) must be smaller than the computed altitude (Hc). Thus the circle of equal altitude for Ho (the one we want to plot) must be farther out from the Ground Position (GP) than is the circle of equal altitude for Hc, -- the smaller altitude angle is farther away, and has the larger radius. Intercept away.



In Diagram B, above, True bearing DR (dead reckoned position) to GP = 150 degrees,(150T).

Here we are using our dead reckoned position (DR) as the assumed position (AP). You will note that it does not fall on a whole degree of latitude as does the assumed position (AP) in Diagram A.

- (See Pages 81 and 84)

Diagram B is merely to show that one can also use the actual dead reckoned position as the assumed position, but this will be the only instance in which we will be doing so. In the remainder of this tutorial we will be using the traditional method of solving the spherical triangle, using a specially chosen Assumed Position (AP) and Sight Reduction tables. (Page 83).

In the above, the observed sextant altitude (Ho) must be greater than the computed altitude (Hc). Thus the circle of equal altitude for Ho (the one we want to plot) must be closer in towards the Ground Position (GP) – Intercept towards -- than is the circle of equal altitude for Hc. The greater altitude angle is closer in towards the Ground Position (GP)

The difference between the observed sextant altitude (Ho) and the computed altitude (Hc) is known as the *"Intercept".*

Using an assumed position, (or in some instances our Dead Reckoned position), as a reference this intercept can place our Line of Position (LOP) either farther away from the Ground Position (GP) --Diagram A above, *(Intercept away)*, or closer in towards the Ground Position (GP) -- Diagram B above, *(Intercept towards)*

--The ***direction*** (*away* or *towards*) in which this intercept is applied along the true bearing line depends on whether the observed sextant altitude (Ho) is greater than the computed altitude (Hc), or vice versa.

--The ***distance*** by which this intercept is applied along the bearing line depends on the difference (converted to nautical miles) between Ho and Hc.

In the diagrams on these pages one may be tempted to assume that where the bearing line intercepts the observed circle of equal altitude (Ho) is, in itself, a two position line navigational fix. It is not. Remember that this bearing line is from an *Assumed Position (AP)*, and while this bearing is usually close to our actual bearing, it is rarely the same. (If we are unable to get a second sighting in order to fix our position with a second position line, we can use this intersection as the *best estimate* of our position).

What we have achieved is to plot on our chart, as a single line of position (LOP), that small relevant part of the circle of equal altitude that equates to our observed sextant altitude (Ho).

The circumference of the circle is so huge that on our chart we can represent our small part of it as a straight line, -- a Line of Position, -- an LOP. Our actual position is usually somewhat to the right or to the left of where the bearing line intersects the circle. However it is somewhere on this line of position (LOP) -- on this part of the circle -- and only by having at least one more Line of Position, -- one more circle -- intersect this first one can we have a navigational fix.

Computing an altitude (Hc) and a bearing from the Assumed Position (AP) is the part of celestial navigation that can involve complicated math, as you will also see below. This math however is made easy for us by the use of pre-computed (Sight Reduction) tables, and becomes merely a matter of adding and subtracting. If, when choosing an assumed position, (AP), we choose one with a whole degree of Latitude, together with a whole degree of Local Hour Angle (LHA), we can enter the tables with whole degrees for these coordinates, making subsequent calculations easier.

The text below relates to the math whereby we determine a computed altitude (Hc) at the Assumed Position (AP). It is not particularly difficult to understand but you may, if you wish, also bypass this for the present. However being aware of what is involved adds to a comprehensive understanding of Celestial Navigation. In practice the math is made easy by the use of pre- computed "Sight Reduction Tables", or by the use of a scientific calculator.

"Sight Reduction" is the term applied to the process of determining a computed altitude (Hc) and plotting a Line of Position (LOP), for an observed sextant altitude (Ho).

Sight Reduction Tables exist that greatly simplify the math involved in finding a Computed Altitude (Hc).

Our objective here is, using some know inputs, to mathematically compute an altitude (Hc) from the Assumed (AP), or reference position, on our chart. This computed altitude (Hc) will be for the same body and for the same time as for our actual observed sextant altitude (Ho).

In doing so we also arrive at a bearing from our assumed position (AP) to the ground position (GP) of the celestial body.

The reason for this computation is to compare and relate this computed altitude, (Hc), computed at a known position on our chart, to our observed sextant altitude (Ho), whose exact position on the chart, while close by, is as yet undetermined.

By comparing these two interconnected altitudes, -- they both have the same ground position (GP) -- and using our Assumed Position (AP) as a reference, we can now plot on our chart as a Line of Position (LOP), that relevant part of the circle of equal altitude corresponding to our observed sextant altitude (Ho)

Since we know the latitude and longitude of our Assumed Position (AP), (we ourselves picked it) and the latitude and longitude (Declination and Greenwich Hour Angle) of the Ground Position (GP) of the celestial body, obtained from the Nautical Almanac for that specific second of time at which we took our observed sextant altitude (Ho), it is possible to mathematically compute the distance in nautical miles between them. **This distance, observer at Assumed**

Position (AP) to Ground Position (GP) of celestial body, is the Zenith Distance (ZD), and to know the Zenith Distance (ZD) (radius of the circle) is to know the computed altitude (Hc): (Diagrams Pages 33 and 39).

Hc = 90 – ZD.

Finding Zenith Distance (ZD), and hence Computed Altitude (Hc), entails mathematically solving a spherical triangle. (Diagram Page 78).

Zenith Distance (ZD) is both an angle and a distance. To know one is to know the other. Zenith distance Angle x 60 = Zenith distance in Nautical Miles.

The three apices of this spherical triangle are also coordinates on the celestial sphere.

However for simplicity they can be projected onto earth as:

(1) P. The nearest pole.

(2) Z. Observer's Assumed Position (AP)

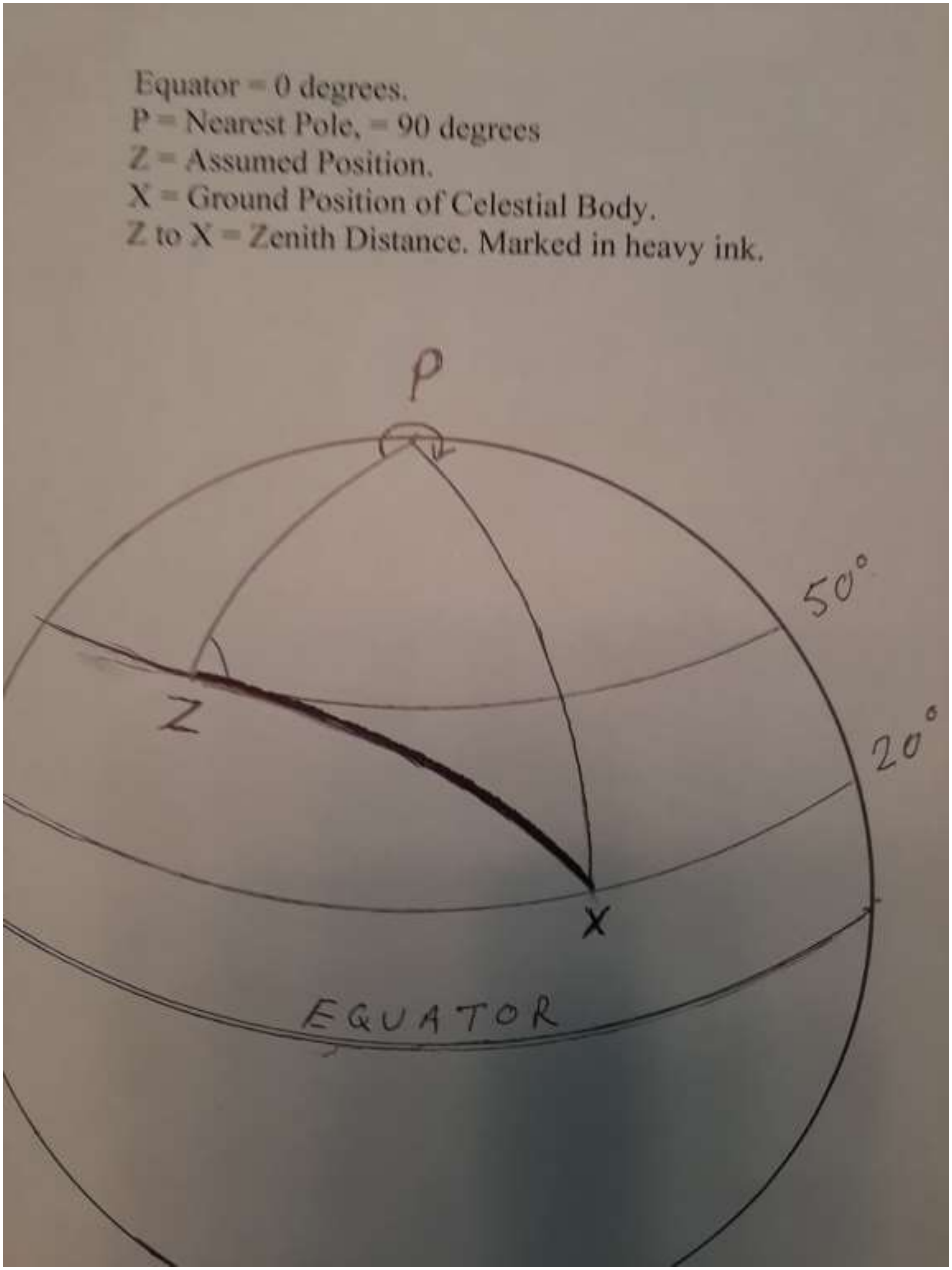
(3) X. the Ground Position (GP) of the celestial body,

Diagram of PZX spherical triangle

(See the diagram below)

This PZX triangle is one of the two classic diagrams used in explaining Celestial Navigation.

The other is the diagram on page 39 showing Zenith Distance (ZD).


Equator = 0 degrees.
P = Nearest Pole, = 90 degrees
Z = Assumed Position.
X = Ground Position of Celestial Body.
Z to X = Zenith Distance. Marked in heavy ink.
P
50°
Z
20°
X
EQUATOR

When we solve this PZX triangle we obtain two of the three pieces of information which we need to plot a line of position (LOP).

(1) -- ***<u>The computed altitude (Hc).</u>*** The length of the side ZX, (in heavy ink above), is observer to Ground Position (GP). This is Zenith Distance (ZD), and from this we arrive at a computed altitude (Hc). (Hc = 90 degrees – ZD.)

(2) -- ***<u>An azimuth angle from which we obtain a true bearing from the Assumed Position (AP) to the Ground Position (GP).</u>*** -- Angle PZX.

[3]**---–** **[**The third piece, which we already know, is the ***Observed sextant altitude (Ho)*****].**

In the above spherical triangle diagram we know the angle at the pole, which is the Local Hour Angle (LHA). It is the angle ZPX. Local Hour Angle is the difference in Longitude between the Longitude of the Assumed Position (AP) and the (Longitude) Greenwich Hour Angle (GHA) of the Ground Position (GP). (In the diagram the difference between the Longitude of Z and the GHA (Longitude) of X*). Local Hour Angle (LHA) is always measured <u>westward</u> from the longitude of the observer.*

So in the above configuration of the spherical triangle, where the Assumed Position (AP), is west of the Ground Position (GP), Local Hour Angle (LHA) is the <u>external</u> angle ZPX. i.e. the difference, <u>measured westward</u> between the Longitude of the Assumed Position (AP), and the (Longitude) Greenwich Hour Angle (GHA) of the Ground Position (GP).

(In actual fact, Sight Reduction tables, (Page 84) entered with either the external Local Hour Angle ZPX or its complimentary internal angle, will still solve the triangle correctly.)

- Since we ourselves pick our Assumed Position (AP) we can select one with a whole degree of Latitude. We can also choose a Longitude which when applied to our Greenwich Hour Angle (GHA) will give us a whole degree of Local Hour Angle (LHA).

LHA = GHA + East Longitude, (of Assumed Position)

LHA = GHA – West Longitude, (of Assumed Position)

By thus selectively choosing an Assumed Position (AP) close to our Dead Reckoned position, but with whole degrees of Latitude and whole degrees of LHA we can make subsequent calculations easier. The Ground Position (GP) of the celestial body, (X in the above diagram), is however determined by the sextant altitude for that particular second of time, and is what it is, and cannot be similiarly manipulated.

We also know the lengths of PZ and PX, and can thus find the length of the third side – ZX, together with the True bearing PZX.

The equator is 0 degrees lat. The Pole is 90 degrees lat.

Latitude can be expressed as an angle north or south of, or as a distance north or south of, the equator.

In solving the spherical triangle, in diagram above, we know the following:

(1) --Side PZ is 90 – (latitude 50) = 40 degrees = 2,400 n.m.

(One degree = 60 nautical miles.)

(2) --Side PX is 90 – Declination (latitude 20) = 70 degrees =4,200 n.m.

(3) –Local Hour Angle (LHA)

Thus, knowing the length of two sides of the triangle, and the enclosed angle, we can obtain the length of the remaining side and the other two angles.

When solved we get the following information;

(1) --The length of side ZX, which is the Zenith Distance (ZD), and from this, (divide by 60), we derive the computed altitude (Hc).

(Hc = 90 degrees – Zenith Distance angle).

(2) -- An azimuth angle from which we obtain a true bearing from the assumed position (AP) to the ground position (G.P).-- In the diagram, the angle PZX.

Thus in addition to obtaining a computed altitude (Hc) we also get a bearing from our assumed position (AP) to the ground position (GP). This bearing we will need later when plotting a line of position.

With respect the True Bearing: In the above configuration of the triangle, where the assumed position (AP) is west of the ground position (GP), the <u>internal</u> angle PZX represents the true bearing of the ground position (GP) from our assumed position (AP). However if the assumed position (AP) were east of the ground position (Last diagram Page 96) the true bearing would be the <u>external</u> angle PZX.

For simplicity, Sight Reduction Tables provide merely an internal azimuth angle and a simple mechanism for converting this angle to a true north bearing “Zn”, for any configuration of the triangle.

Thus, we need not involve ourselves in the complicated math relating to spherical triangles but can solve this triangle quite simply by the use of what are known as **“Sight Reduction Tables”** one of which is a publication known as “Ho 229”for marine use. Another is Ho 249 used by the Air Force. Either can be used for sight reduction whether at sea or in the air. Since we ourselves select our Assumed Position, (AP) we can select this position so as to enter the tables with a whole degree of latitude for the assumed position (AP), and a whole degree of LHA, making calculations easier.

Entering the tables with:

(1) Latitude of the Assumed Position (AP), (whole degree)

(2) Declination (Latitude) of the Ground Position (GP) (Degrees, minutes and tenths of minutes).

(3) The Local Hour Angle. (LHA) (whole degree)

We obtain both Hc (computed altitude) and we also derive the bearing from Assumed Position (AP) to Ground Position (GP).

The spherical triangle may also be solved using a scientific calculator.

- ❖ If not using sight reduction tables, but using either a scientific calculator, or laboriously solving the spherical triangle mathematically without a scientific calculator, the Dead Reckoned Position (DR) itself can be used as the Assumed Position (AP). (Diagram B Page 73).

In that instance, full coordinates of both Local Hour Angle (LHA) and Latitude of the Dead Reckoned Position, i.e. Degrees, Minutes and tenths of minutes, would, of course, need to be entered.

While not essential, knowledge of what the PZX triangle consists of, and of what is involved in solving this spherical triangle, by whatever method, is useful and rewarding in the overall understanding of celestial navigation.

In general there are two methods of solving the spherical triangle, in order to retrieve the two pieces of information in the equations below:

(1) Tabular. By the use of pre-computed sight reduction tables. This is the traditional method.

(2) Formula. By the use of scientific calculators, or computer programs. A variety of these are available.

And for the Maschochistic Mathematician (MM!), who wishes to solve this longhand, below are the two mathematical formulae used to retrieve these two pieces of information from the spherical triangle:

(1) -- Computed alt.(Hc) = Cos (ZX) = [Cos(PZ) . Cos(PX)] + [Sin(PZ) . Sin(PX) . Cos(ZPX)]

(2) – Bearing to (GP) = Cos PZX = [Cos(PX) – Cos(ZX) . Cos(PZ)] / [Sin(ZX) . Sin(PZ)]

Here also is a link to this.

https://astronavigationdemystified.com/calculating-azimuth-and-altitude-at-the-assumed-position-by-spherical-trigonometry.

Sight Reduction Tables either H0 229, or Ho 249, entered with knowledge of the Local Hour Angle (LHA), (whole degrees), the latitude (whole degrees) of the Assumed Position (AP), and the exact degrees, minutes and decimal minutes of Declination (Latitude) of the Ground Position, (GP) pre-computes, and greatly simplifies, for us the math for obtaining a computed altitude (Hc).

Again, in addition to calculated altitude (Hc) we also obtain an azimuth angle from which we derive a True Bearing from Assumed Position (AP) to Ground Position. (GP).

Here is a resource explaining Sight Reduction Tables:

https://www.youtube.com/watch?v=1oim2u71KCA

[The nautical almanac, first published in 1766, and now published annually, is a different publication to the Sight Reduction Tables and is consulted when using a sextant to take a sight on a celestial body. The nautical almanac gives us -- among many other things -- the Ground Position (G.P.), in Declination (Latitude) and Greenwich Hour Angle, GHA, (Longitude), of the Sun, Moon, the 4 navigational planets and the 57 stars, (extra step of adding Sidereal Hour Angle to GHA of Aries, for these 57 stars) used in Celestial Navigation interpolated for every second of every day of the year. One enters with date and Greenwich Mean Time. -A note of caution with respect entering with the correct Greenwich *date,* -- for example 2200 May *6th* at longitude 150 degrees west is 0100 May *7th* at Greenwich.]

Information in the Nautical and Air almanacs, and in the Sight Reduction Tables is tabulated in Degrees minutes and tenths of minutes of arc, with respect position, and tabulated in Hours minutes and seconds, with respect time.

Here is a resource explaining the Nautical Almanac:

https://www.youtube.com/watch?v=GLxsa91cySA

Further examples in Plotting a Line of Position

Lines of Position (LOP) would be plotted on a plotting sheet overlain with a Latitude and Longitude grid.

Let's again take an example in order to plot our Line of Position (LOP). (Diagram, Page 88).

Observed sextant altitude (Ho)

= 40 degrees 36 minutes (minutes of arc).

Computed altitude (Hc) at Assumed Position (AP)

= 40 degrees 30 minutes (minutes of arc).

True Bearing from Assumed Position (AP) to distant Ground Position (GP) = 140 degrees. (140T)

Comparing these two altitudes we see that the observed sextant altitude (Ho) is the larger angle and must thus be closer in towards the Ground Position (GP) than is the computed altitude (Hc) – “Intercept towards.” The comparison also tells us that it is closer in by 6 minutes of arc, which is 6 nautical miles.

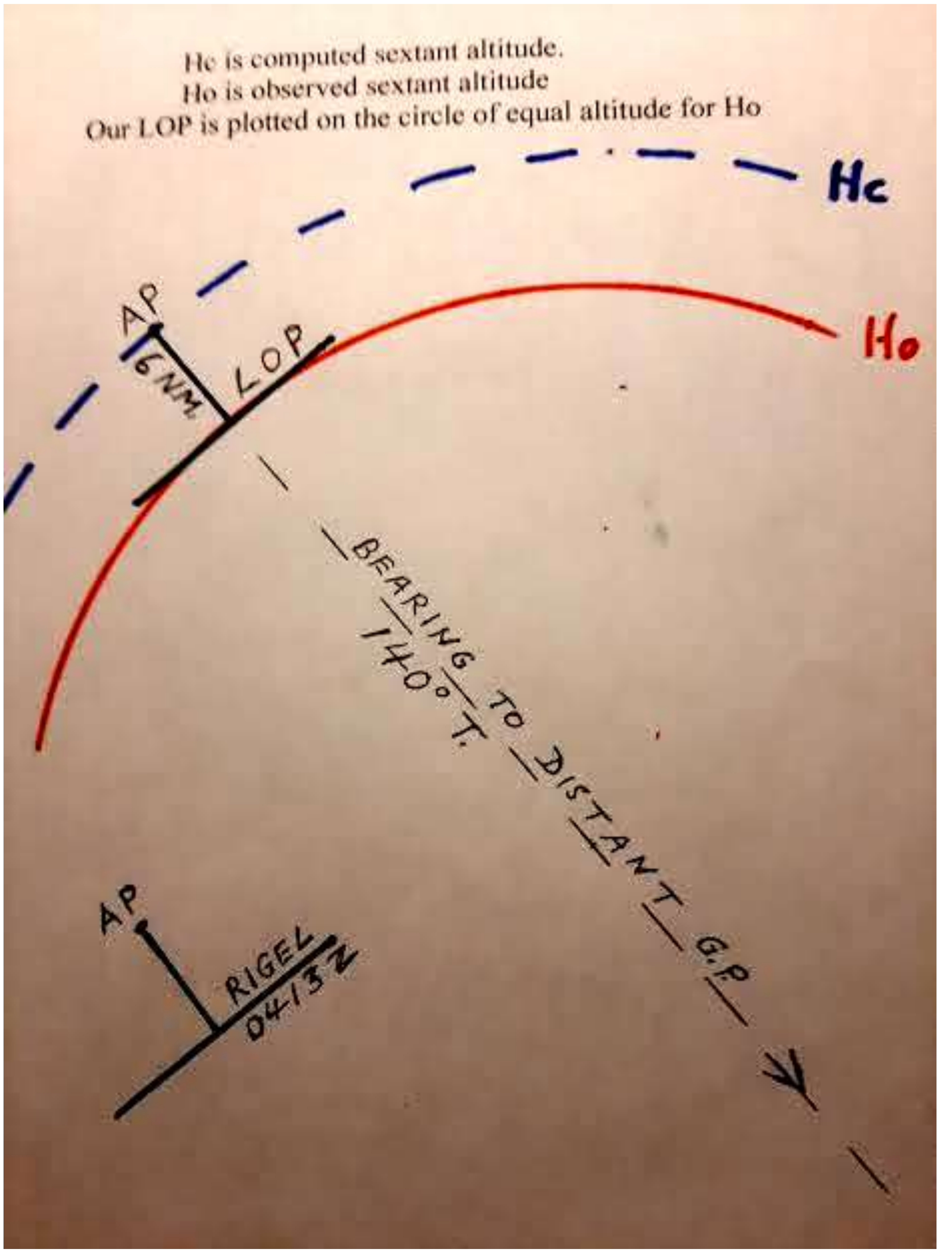
Firstly, through our Assumed Position (AP), we mark out the bearing, 140 degrees, towards the distant Ground Position (GP) of the celestial body.

The circle of equal altitude represented by our observed sextant altitude (Ho) is the one we want to be on and is the one we wish to chart. We now know that it is 6 nautical miles closer in towards that distant ground position than is the circle of equal altitude for our computed (Hc), and we know that the circle for our computed altitude (Hc) passes through our Assumed Position (AP).

From the Assumed Position (AP) We now mark off 6 nautical miles along our bearing line towards the distant ground position. At this point on the bearing line we draw a line at right angles to the bearing line and this is our **Line of Position**.(L.O.P.) (See diagram below on Page 87).

Again the larger diagram below on page 88 is for explanatory purpose only.

In the actual plotting of a line of position only that part of the bearing representing the intercept would be plotted, and neither of the two circles of equal altitude would appear as such. However our eventual Line of Position (LOP) would represent as a straight line that small relevant part of the circle of equal altitude for observed sextant altitude (Ho). The smaller image to the lower left of the diagram is a representation of what the plotted line of position would look like.



This then is our first line of position. We have plotted on our chart that small arc of the circle of equal altitude that corresponds to our observed sextant altitude (Ho). At least one more circle, preferably two more, and we have a fix.

To summarize

(And forgive the repetition!).

Since altitude of celestial body equates to distance, by comparing these two altitudes, (observed and computed) we are also comparing their distances from the Ground Position (GP) of the celestial body,-- ***we are comparing the radii of two concentric circles.***

However it is easier to compare angles, and we need only to compare the angular *difference* converted to nautical miles.

It is only necessary to compare the *differences* between these two altitudes, Ho and Hc. By doing so we arrive at the same figure we would get if we had compared the differences between their zenith distance (complementary) angles, or the differences in their radii, distances in nautical miles from that distant ground position (GP)

The comparison tells us two things.

It tells us:

(1) Whether our circle of equal altitude for our observed sextant altitude (Ho) is closer in towards, or farther away from, the Ground Position (GP) than is our circle of equal altitude for our computed altitude (Hc).

(2) By converting degrees and arc minutes to nautical miles, it tells us by how much.

We know where one segment of a circle of equal altitude, the one corresponding to the computed altitude (Hc), lies on our chart. Where, does the other circle segment, the one corresponding to our observed sextant altitude, (Ho) lie on our chart? Where is Ho in relation to Hc?

Now, -- if we wish, and for demonstration purposes only --we can plot that part of the circle of equal altitude that equates to our computed altitude (Hc) as we know it must pass through our Assumed Position (AP) and it will be at right angles to the bearing line that points to the ground position. This arc of the circle would be our Line of Position (LOP) *had we been at our assumed position.*

Depending on which altitude --Ho or Hc – is the greater in degrees, it will place us either closer in towards the Ground Position (GP) or farther out from the Ground Position (GP) than is the Assumed Position (AP). Our arc of the circle of equal altitude for our observed sextant altitude (Ho) will have to be adjusted accordingly by the amount of the difference, between the two altitudes, -- computed (Hc) and observed (Ho). *Our Assumed Position (AP) is the reference point <u>on our chart</u> from which we can make that adjustment.*

When we plot our first Line of Position (LOP) it gives us only the information that we are somewhere along that line. That line is actually a small part of the often vast circle of equal altitude that we have now been able to plot on our chart.

Where further lines of position (circles of equal altitude) intersect would be our navigational fix (Page 44). Where two such lines of position intersect will give us a good idea of our position, three is better. Three such lines rarely intersect at a common point but usually close to a common point, resulting in a small triangle referred to as a “cocked hat.” The center of this “cocked hat” is taken as the navigational fix. The smaller the “cocked hat” the better the accuracy. A navigational fix establishes both our latitude and our longitude.

To increase accuracy it is preferable that Lines of Position (LOPs) cross each other as close to 90 degrees (two LOPs), or 60 degrees (three LOPs), as possible.

Celestial navigation is both an art and a science.

The marine sextant is graduated to an accuracy of 0.1 minutes of arc, which is one tenth of a nautical mile (607 feet). In no way however, does this mean that in practice such accuracy is achievable in celestial navigation.

Unavoidably there are a number of inherent inaccuracies in each stage of the process, -- in taking the sextant sight, in sight reduction, and in plotting.

Note. *(Google, Chris Kreitlein’s “Accuracy limitations in Celestial Navigation.”)*

In celestial navigation, at sea or in the air, fixing one’s position to within 5 nautical miles is considered acceptable accuracy.

Those who claim that they can consistently fix their position to within an accuracy of less than half a mile are either consistently lucky, or consistently lying!

If any Intercept approaches, or is greater than one degree (60 nautical miles), and if other sources of error are ruled out, the accuracy of the Dead Reckoned (DR) position should be reevaluated. Similiarly if the position of a fix obtained approaches, or is greater than 60 nautical miles from where an intercept intersects the LOP, the accuracy of the fix is suspect. The sight should then be reworked for greater accuracy, using that first fix as the new Dead Reckoned position.

When we use a sextant to measure altitude of a celestial body in order to obtain a navigation fix, we record three pieces of information:

(1) -- From the sextant. -- Sextant altitude (Ho), which we put aside for future comparison with computed altitude (Hc).

(2) -- From our watch: -- The exact Greenwich Mean Time (GMT) of the sextant reading.

(3) -- From the Nautical Almanac. -- Ground Position (GP) in Declination and GHA (Latitude and Longitude) of celestial body at that particular time <u>to the second</u>. This latter will be one of the three inputs needed to compute an altitude (Hc) at the Assumed Position (AP). This computed altitude (Hc) will then be used to compare to the actual observed sextant altitude (Ho), in order to plot on our chart an actual Line of Position (LOP)

<u>Running fix</u>

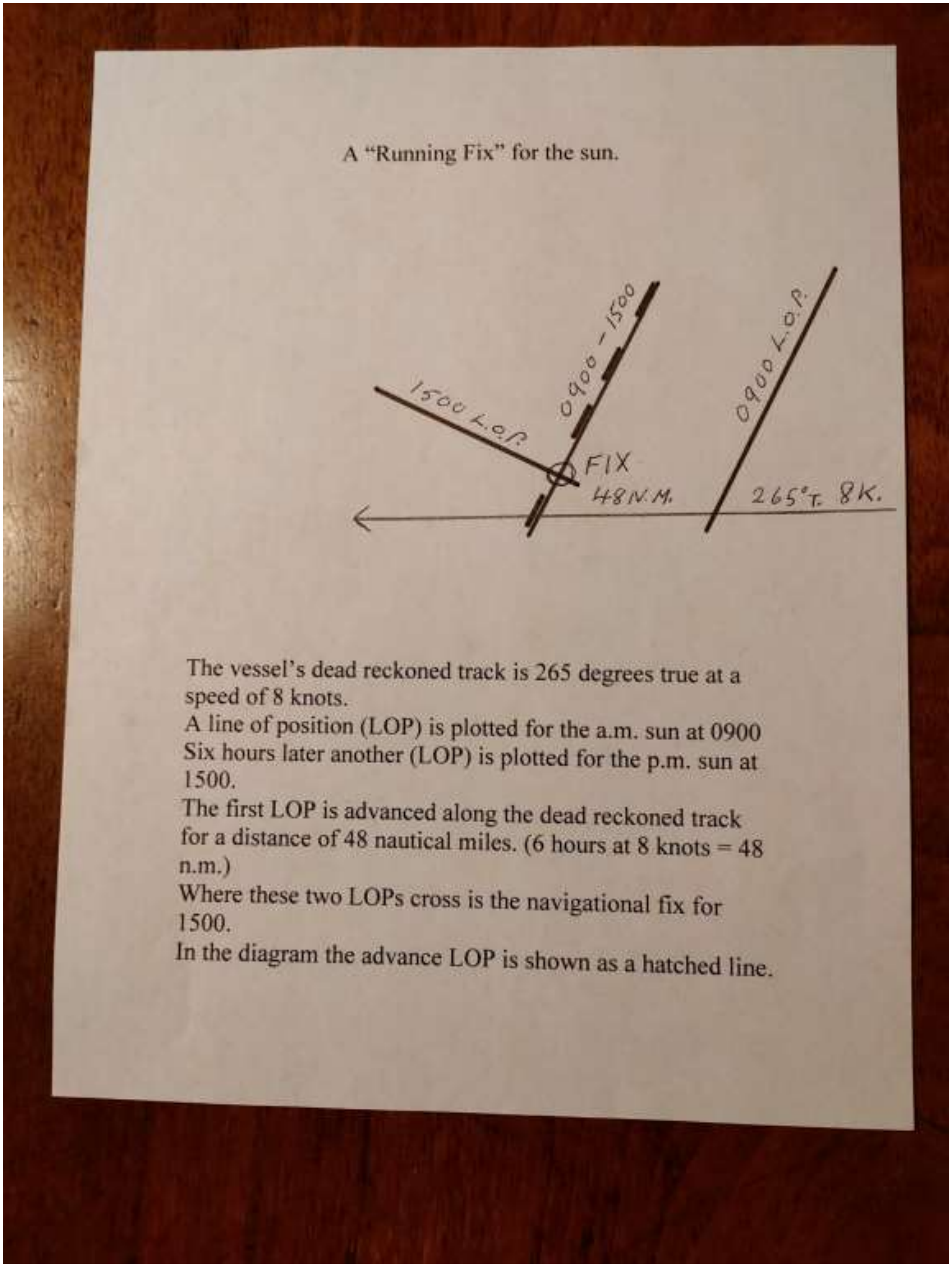
If the vessel has proceeded on its course between sightings, then earlier Lines of Position (LOPs) must be “advanced” to intersect the most recent, thus obtaining a navigational fix.

LOPs are advanced along the Dead Reckoned (DR) course taking into account the vessel's speed and the time elapsed, together with effect of wind and tide, if known and if applicable.

This applies to sightings on two or more different celestial bodies at differing times. It also refers to the same body with sightings at different times. Sightings on the same body after a considerable time lapse are referred to as a "running fix"

On Page 94 below there is a diagram of a "running fix" on the sun. The sun's bearing for the 0900 line of position (LOP) is to the south east. The sun's bearing for the 1500 (LOP) is to the south west. Since the bearings are close to ninety degrees apart, so too the lines of position, (which are at 90 degrees to the bearing lines), will be close to ninety degrees apart.

Shooting the sun in the a.m. and p.m. and advancing the a.m. LOP is a particularly useful navigational technique. It is usually also accompanied with a noon latitude estimation. Latitude by the noon sun, a mainstay of celestial navigation, is discussed later on page 96.



A "Running Fix" for the sun.

The vessel's dead reckoned track is 265 degrees true at a speed of 8 knots.
A line of position (LOP) is plotted for the a.m. sun at 0900
Six hours later another (LOP) is plotted for the p.m. sun at 1500.
The first LOP is advanced along the dead reckoned track for a distance of 48 nautical miles. (6 hours at 8 knots = 48 n.m.)
Where these two LOPs cross is the navigational fix for 1500.
In the diagram the advance LOP is shown as a hatched line.

6 hours have elapsed between 0900 and 1500

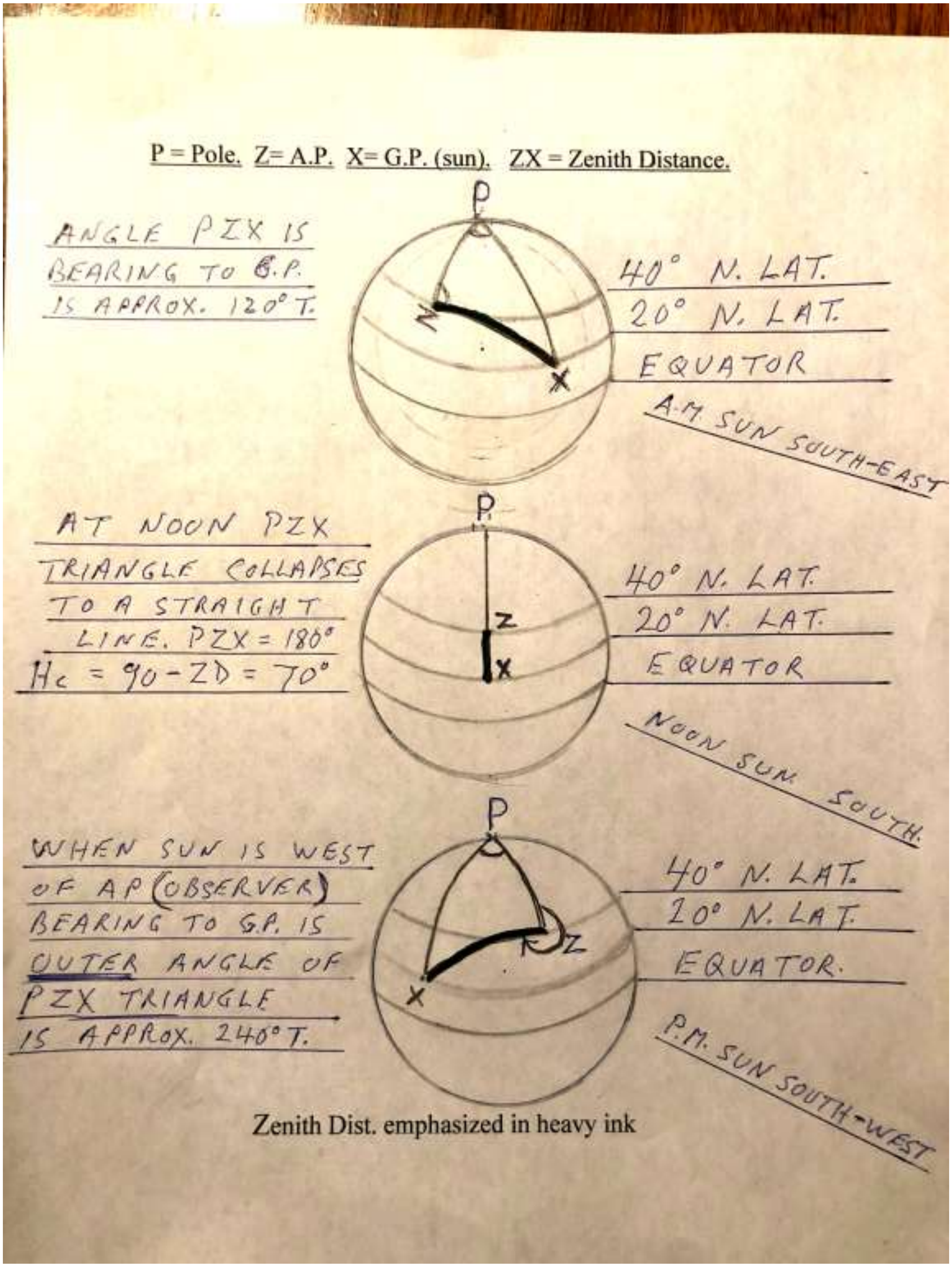
6 hours at 8 knots equals 48 nautical miles

The earlier LOP at 0900 is advanced 48 nautical miles, along the vessel’s course, maintaining its directional orientation.

The accuracy of the vessel’s estimated course and speed between sightings affects the accuracy of the navigational fix.

More recent LOPs can also be retarded, to obtain a navigational fix for the time of the earlier sighting.

Below are 3 diagrams of the Sun (x) moving westward across the sky in the Northern Hemisphere.



Zenith Dist. emphasized in heavy ink

Part 3. Latitude and Longitude

Latitude is distance in degrees north or south of the equator.

The equator is 0 degrees latitude. Either pole is 90 degrees latitude.

--Latitude by Polaris, **in the northern hemisphere**.

It is the sextant altitude (Ho) of Polaris -- with adjustment for the fact that earth's axis does not quite point directly at Polaris.

Polaris, visible in the northern hemisphere, is currently about three quarters of one degree offset from True North.

Failure to compensate for this offset can result in a latitude error of up to 45 nautical miles.

Latitude by the noon sun, is 90 – sextant altitude (Ho), -- with adjustment for the sun's declination.

The sun at noon is either due north, due south, or, (possible in the Tropics) overhead the observer.

Finding latitude by the noon sight -- meridian passage, when the Sun and the observer are on the same meridian of Longitude -- is a special circumstance, and a much simpler procedure than fixing one's position in both Latitude and Longitude by using the intercept method. Latitude by the noon sun does not require accurate time, as such, and it does not involve solving any spherical triangle, because the spherical triangle collapses into a straight line (See middle picture on page 95 above), and spherical trigonometry is reduced to plain arithmetic, when the the observer sights the sun at its highest point in the sky.

One measures the sun's altitude *at its highest point of arc* across the sky, i.e. the noon altitude of the sun, The sun is at that time on your meridian of longitude and, depending on where you are on the earth's surface, is either directly overhead or due north or due south of you.

Using a nautical almanac one finds the sun's Declination, (Latitude north or south of the equator, i.e. where the sun is on its passage from one solstice to the other) on that particular day. It is then a simple matter of addition or subtraction depending on one's position relative to the sun and the equator. The sun's declination can also be calculated by a formula.

For simplicity let's say that we get a sextant altitude of 30 degrees <u>when the sun is on the equator.</u> (which only happens twice a year, at the time of the equinoxes)

Ships latitude is then merely 90 -30 = 60 degrees.

At all other times however, the sun's declination has to be applied to the above Zenith Distance (ZD) to obtain latitude.

Zenith distance (ZD), 90 – Sextant altitude (Ho) is important in determining Latitude, -- as it is in many other aspects of celestial navigation.

--*Ship's Latitude* is ship's distance in degrees north or south of the equator.

--*Sun's Latitude* (Declination) is sun's distance in degrees north or south of the equator.

--Zenith Distance, [90 – Sextant altitude (Ho)] is distance between *ship and ground position (GP) of sun*.
--For our latitude we need to know distance between *ship and equator*.

There are three possibilities with respect the relative positions of sun, ship and equator.
(See diagram Page 101)

(1)—The Sun is between the ship and the equator.
Ship's Latitude = (90 - Ho) + Sun's Declination

(2)---The sun and the ship are in different hemispheres.
Ship's Latitude = (90 - Ho) – Sun's Declination

(3)---Ship is between the Sun and the equator. (Possible in the Tropics where, depending on time of year, the sun can be as far north or south of the equator as 23.5 degrees.)
Ship's Latitude = Sun's Declination – (90 – Ho)

Usually, if sailing in the same hemisphere and not entering or leaving the tropics, only one of the above will be applicable for all of the voyage, but if in doubt simply draw a diagram as below:

Zenith Distance (ZD) (90 – sextant alt.) ship to sun is marked in heavy ink below.

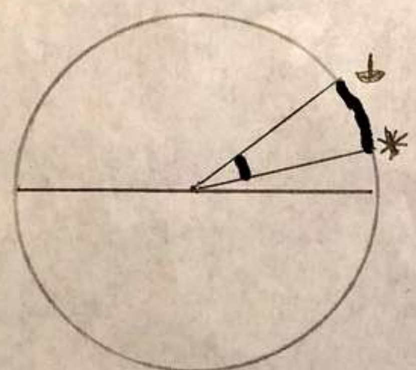
Ship's latitude is the angular distance of the ship icon, in the diagram below, from the equator.

.

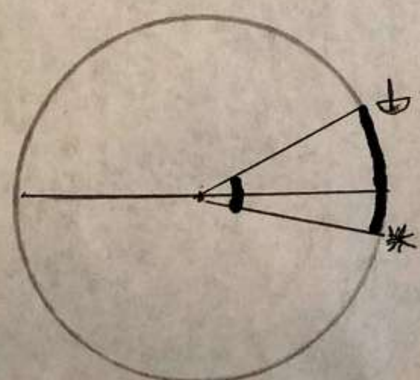
4

Remember that 90 – Ho = Zenith Distance, (ZD) = distance between ship and Sun. In these diagrams this angle and distance are both accentuated in heavy ink.

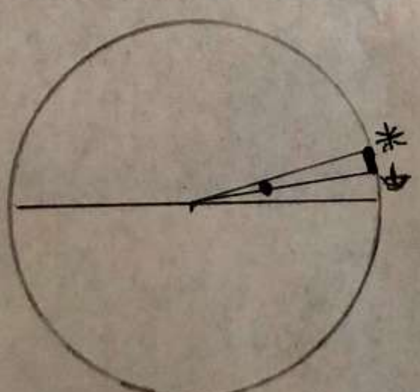
Remember Latitude of the ship and Declination (Latitude) of the sun is an angle measured at the center of the earth and represented on the earth's surface as an angle and a distance north or south of the equator.



Lat = ZD +Declination.

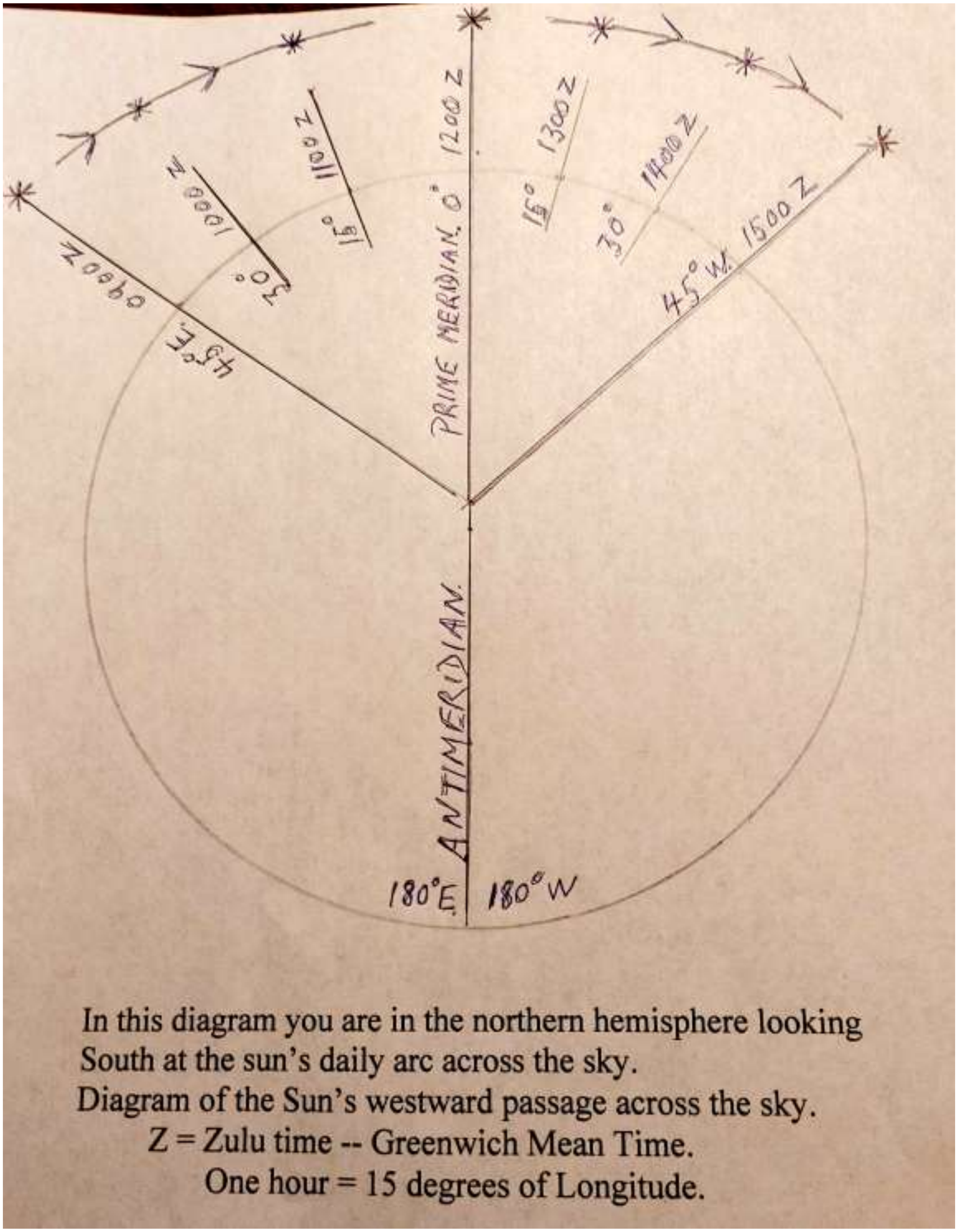


Lat =ZD - Declination



Lat = Declination – ZD.

Diagram of Longitude



In this diagram you are in the northern hemisphere looking South at the sun's daily arc across the sky.
Diagram of the Sun's westward passage across the sky.
Z = Zulu time -- Greenwich Mean Time.
One hour = 15 degrees of Longitude.

Longitude is a measurement East or West of the Prime Meridian.

Longitude by chronometer is the difference in time (converted to degrees) between the noon sun at Greenwich and the noon sun at observer's position.

- *But see **EoT**. Page 106 (below)*

One hour = 15 degrees.

Establishing one's Longitude by the noon sun involves determining accurately the *time* of an *event.*

(The *event* is the sun on the observer's meridian of longitude. This is the (apparent) sun at its highest point on its daily passage across the sky. This is observer's noon, or Local Apparent Noon (LAN)

The *time* is the Greenwich Mean Time at which this *event* occurs at the observer's (position) longitude.

[***Note, as an aside***: By comparison, establishing **Latitude** by the noon sun involves merely determining the *sextant altitude* of that event. The event being the sun at its highest point in the sky, and as such, for latitude, accurate time, while helpful for convenience, is not essential.]

The mean sun is over the Greenwich Meridian,-- O degrees longitude,-- at 1200 hours (noon, Greenwich Mean Time)

Say our watch, set to GMT, reads 1430 hours *(Time)* when we observe the sun on our meridian of longitude *(Event)*

The mean sun moves westward at a rate of 15 degrees every hour

If our watch, set to Greenwich Mean Time, reads 1430 when we observe noon at our meridian of longitude, the sun has moved two and one half hours -- 37 degrees and one half degrees -- westward away from noon at the Greenwich Prime Meridian.

Our Longitude would be 37 degrees 30 minutes (of arc) West.

Had our watch, set to GMT, read 0900 hours, when we observed the sun on our meridian, it would still be three hours -- 45 degrees-- before the sun reached the Prime Meridian and we would have been at longitude 45 degrees East.

[**Note**: These two examples above are given as a simple illustration of the concept, *but would only apply on the four occasions in the year when mean and apparent sun are both at the same place in the sky,*- when both mean and apparent suns are both over the Prime Meridian together at 1200 noon.

On all other occasions one must apply the **Equation of Time (EoT)** when finding Longitude.]

Note: *[Enter "Longitude by Chronometer" in any search engine and you will be amazed how many times the above over-simplification is put forward as a comprehensive explanation of Longitude, without any mention of the need to apply the EoT. Failure to apply the EoT could lead to an error in Longitude of as much as four degrees at certain* *times of the year, when the maximal value for the Equation of time can be around 16 minutes. (Diagram Page 19).*

An error of 16 minutes of time would equate to an error of four degrees in Longitude. This would equate to an error of 240 nautical miles at the equator, or an error that could place the Prime Meridian running through eastern France, rather than eastern England!

Not applying the EoT would result in every location on earth having a constantly shifting Longitude day by day throughout the year].

- **(EoT) Equation of time explained**

A factor, which has to be applied in determining longitude by noon sighting is something called the *Equation of Time.* (EoT)

The *Mean* in Greenwich Mean Time relates to the fact that in order for the sun to be always exactly on the Prime Meridian at 12 noon for every day of the year, we had to average it's varying speed around the earth over the year. It is this averaged sun that moves around the earth at 15 degrees per hour. However the apparent, or true sun, which we are measuring at our longitude does not move at a uniform speed, it moves slightly faster some times of the year slightly slower at other times of the year. This apparent sun can be farther to the east or farther to the west than is the mean sun on any particular day. Only on four days in the year, when the values for the equation of time are zero, is this apparent sun over the Greenwich Meridian at 1200 noon. It is this apparent sun that we are timing on our meridian. When comparing the time difference between when the mean sun is over the Greenwich meridian and the apparent sun is over our meridian we cannot meaningfully compare *Mean* (averaged) sun at Greenwich to *Apparent* (true) sun at our meridian of longitude. On most days the mean sun and the apparent sun are at different places in the sky. We must compare apples to apples. By applying the Equation of Time we reconcile *Apparent* Solar time with *Mean* Solar Time.

Written as an equation;

EoT =Apparent (true) Solar Time – Mean Solar Time.

If Apparent Solar time is ahead of Mean Solar time the sign is positive, if behind the sign is negative.

The following is the simplest method of applying the Equation of Time, when finding Longitude.

Let's say that with our watch set to Greenwich Mean Time, (GMT) we observe the noon sun on our meridian at 1332:00. We enter the nautical almanac for that day and under "*the Equation of Time"* find the Mean time of Meridian Passage at Greenwich listed as 1202. to nearest minute. Subtracting 1202 from 1332:00 gives a time difference of 1 hour 30 minutes, -- which gives us our correct longitude of 22 degrees 30 minutes West.

Longitude is west since our time of observation was after Greenwich noon.

By applying the Equation of Time we are comparing apples to apples. We are comparing the difference in time between the mean sun at Greenwich and the mean sun at our location. The time of both events is Greenwich Mean Time. There is a mechanism whereby one can gain more accuracy by interpolating for seconds of time for Meridian Passage at Greenwich, but the above is a commonly used method of applying the Equation of Time.

Had we, in the above example, made the wrong assumption and used *1200* rather than *1202* as our Greenwich reference time we would have been in error by 2 minutes of time which in this instance would have placed us one half degree (30 minutes of arc) of longitude farther west.

Let's take another example and assume that this time, again with our watch set to Greenwich Mean Time (GMT), we observe the noon sun at our meridian at <u>0856:00</u> We enter the Nautical Almanac and find that for that day, time of Meridian Passage at Greenwich is 1156. To find our longitude we subtract 0856: 00 from <u>1156</u> to get <u>three hours</u> which would represent our <u>correct</u> Longitude of <u>45 degrees East</u>.

Longitude is east since our time of observation was before Greenwich noon.

Again had we used *1200* rather than *1156* as our Greenwich reference time we would have been in error by four minutes of time which would be an error of one degree farther east.

Determining longitude is easier when our watch (or one watch in our possession) is set to read Greenwich Mean Time. Alternatively we can convert our watch time in our time zone to Greenwich Mean Time (GMT).

Thus we see that with accurate time the noon sight can, in addition to our latitude, give us also our longitude,-- but not to the degree of accuracy that we can achieve for latitude.

--The *time* (more difficult to get exact) at which the sun is on our meridian gives us Longitude.

--The *sextant altitude* (easy to determine the highest point of arc, when the sun is due north or due south or directly overhead.), gives us Latitude.

Longitude by the noon sight is less accurate than is Latitude, for the following reason.

--It can be difficult to determine the exact *time* when the sun is at it's daily highest point in the sky, because at noon, at the top of its arc, the sun seems to hang in the sky for a short period. (There is a mechanism we can use that helps to increase accuracy) The sun is travelling westward so fast that one minute of error in time would result in one quarter of a degree (15 minutes of arc) error in Longitude. In determining Longitude by the noon sighting it is often difficult to achieve an accuracy within 10 nautical miles.

Part 4 Navigation in the Past.

Prior to the mariner's ability to establish Longitude, estimation of Latitude alone was still a valuable aid to navigation.

The navigator, if he knew the latitude of his destination, would sail to intercept that latitude and then sail, east or west, along that parallel of latitude to his destination.

He obtained, and maintained, the desired latitude by daily sightings of the sun at noon, and Polaris by night, (often twilight, to obtain a visible horizon).

This was referred to as, "*Running down the latitude.*"

By thus maintaining a constant latitude the mariner had also the advantage of maintaining a constant heading. However a disadvantage on long voyages would be that the distance travelled would be farther, by comparison to the the present method of following a great circle route. However maintaining a great circle route would also involve constant, (or a least frequent), heading changes.

The marine sextant was not invented until the mid 1700s, and the first publication of the Nautical Almanac was around the same time.

Prior to that time however more rudimentary methods existed for finding Latitude by Sun and Polaris. As early as the 1500s tables of the sun's declination, (position north or south of the equator) existed, and a Polaris correction was also known. (Since Polaris is not directly aligned with the earth's axis a correction is needed in finding Latitude by Polaris)

One mechanism used in those days to allow for the fact that Polaris does not exactly coincide with Celestial True North was known by the archaic terminology; *"Regiment of the Pole."* It involved using the relative positions of the trailing stars of two nearby constellations, Alkaid in Ursa Major, and Segin in Cassiopia, as a means of establishing the offset of Polaris from True North. In Columbus' time this offset would have been three and one half degrees. Currently it is three-quarter of one degree.

As an example, Spanish or Portuguese sailors, sailing to, say the West Indies in the 1500s,1600s and early 1700s, would sail in a southerly direction to intercept the 21st parallel of latitude on which Cuba lay. Aided by the easterly trade winds in those latitudes they could not fail, (barring the many perils facing "they that go down to the sea in ships") to make landfall somewhere in the West Indies, hopefully Cuba.

Returning they would sail to the northeast to avail of the westerly winds in the more northerly latitudes, perhaps turning east to run down the 38th parallel of latitude, on which both the Azores and Lisbon lay. The Azores, a group of islands in the mid Atlantic around one thousand miles west of Portugal, had been discovered by Portuguese sailors in the early 1400s, -- some evidence suggests 700 years earlier by the Vikings!

However, this reliance on "shipping lanes" was a boon, not only to legitimate mariners, but also to pirates lying in wait to plunder those ships laden with treasure; treasure that had been, in turn, plundered from the New World.

In today's world of pinpoint GPS accuracy it is difficult to relate to the hazards and uncertainty of navigation in those days.

An example of the difficulty in establishing Longitude in those days is demonstrated by an incident on Columbus' first voyage of discovery of the new world. Returning to Spain in 1493 his two remaining ships were caught up in a violent storm. Sighting land, which turned out to be the island of Santa Maria, the southernmost island of the Azores, they sought shelter there.

"Some thought the land sighted was the Madeiras",

-- This would represent not only a longitude error of 9 degrees, or around 470 nautical miles, but also a latitude error of 5 degrees, or 300 nautical miles!

"Others thought the land sighted was the coast of Portugal,"

This would represent an even greater Longitude error of 16 degrees; in those latitudes, a gross error of 800 nautical miles!

What would not these explorers have given for the ability to establish their position to within an accuracy of 5 nautical miles on a vast ocean, by using observations of the heavens!

Only Columbus himself estimated the land sighted to be an island of the Azores archipelago.

Columbus' voyage was sponsored by Spain, and in those days Spain and Portugal were political rivals. The Islands of the Azores were a Portugese colony, and normally Spanish ships would avoid Portugese territory, but it was literally a case of any port in a storm. In the event some of Columbus' men were put in prison when they landed on Santa Maria, but were subsequently released.

Columbus had one further unwanted contact with Portuguese authorities, when again storms drove him to end his voyage at Lisbon in Portugal, rather than at his home port of Palos de la Frontera on the Spanish mainland.

In the early 1400s Portuguese and Basque sailors ventured far from their homelands, on voyages of exploration.

“Prince Henry the Navigator” of Portugal, who died in 1460, was a noted patron of maritime exploration; although he himself never ventured far “afield”. He founded a school at Sagres on the south west tip of the Iberian Peninsula dedicated to promoting exploration and to furthering knowledge of navigation. Prince Henry encouraged and aided exploration westward into the Atlantic and southward down the African coast, even as far south as the dreaded Cape Bojador. Prior to the 1400s no one, other than a Carthaginian in 500 b.c.! had ever ventured beyond this African Cape, for “*beyond here be monsters.”* Cape Bojador, a protuberance on the coast of west Africa, was a graveyard of ships. Rocks, shoals, gales, mists, - and superstition, had given the cape a fearsome reputation. Cape Bojador was a physical and a psychological barrier to exploration of the coast of West Africa.

Gil Eannes, a name now largely forgotten, eventually sailed beyond Cape Bojador, opening the route to Asian trade that others would follow around the southern tip of Africa. Sadly this also opened a route to the African slave trade!

The Basque too, like the Portugese, were noted early explorers.

Basque sailors were aboard the Santa Maria with Columbus in 1492. A Basque, Juan de Elcano sailed Magellan’s ship, the “Victoria” home from the Philippines, (after Magellan was killed there), completing the first circumnavigation of the globe.

Basque fishermen had reached Newfoundland and the Gulf of St. Lawrence by the early 1500s.

On land too the Basque strayed far from home.

Basque shepherds tended sheep on the pampas of south America and in mountain meadows in California.

Once, when hiking in a remote area of the Chilcotin Mountains, west of the Fraser River in B.C. I came across a small hut, whose roof and log walls were in advanced decay. The derelict hut, half hidden in a copse of pine trees, overlooked a vista of alpine meadows, which in early Summer were ablaze with a glorious profusion of alpine flowers. I subsequently learned that it had been the abode of a Basque shepherd in the early 1900s. Each Summer he had brought his sheep to graze in those high mountain meadows. Here in his lonely solitude, he and his dogs would have guarded his flock from bears, (of both the grizzly and the black variety), from wolves, coyotes, bobcats, wolverines, foxes, and from eagles overhead. His only access to the area was by way of a small cable ferry across the mighty Fraser river, at a place called Big Bar. This type of ferry, known as a reaction ferry, uses the power of the current to propel it across the river on a cable. The log hut was 30 miles from the ferry crossing, and from there a further distance of 25 miles to the nearest town, -- a long hike out for help should he fall prey to accident or illness.

The Basque said of themselves;

"God gave us a small country for our birth, -- and the world to die in."

The same might be said of many a small country whose progeny are scattered across the oceans of the world.

I wonder if that shepherd ever made his way back to the land of his childhood; to wander with an old man's memories in the solitude of his own mountainous land. Or is he buried in an overgrown and forgotten grave, an ocean and a continent away from the land where he experienced his first childhood memories.

Others too sailed far from home.

As early as the time of the American Revolution, towards the late 1700s, New England whaling ships were operating off the coast of Brazil and even at latitudes as far south as the Falkland Islands.

Some unlettered, self-taught ship's captain, *"some mute inglorious Milton,"* whose first experience of the sea may well have been as a 12 year old cabin boy, had safely navigated his vessel 6,000 hazardous sea miles from Nantucket to the whaling grounds in the South Atlantic! Once there he and his ship's crew would engage in the even more hazardous pursuit of the whale. Over a year later, homeward bound, and ballasted with 2,000 barrels of prized whale oil to light the lamps of New England, he'd have watched the Southern Cross sink nightly lower astern. Shortly after crossing the equator the welcome sight of Polaris rising from the sea, would beckon him northward. When, about a month after crossing the equator, sun by day and star by night confirmed a latitude of 41 degrees 18 minutes north he would turn his ship to the west to run down the latitude towards his island home, -- as yet an unseen speck on the ocean below that western horizon.

After such a long voyage he would know that longitude by dead reckoning was likely to be extremely unreliable. Most likely he would not have had the luxury of a reliable chronometer, which in those days would have been both expensive and hard to come by. However when the ocean depth beneath his keel changed abruptly from thousands of feet in the open Atlantic to hundreds of feet over the continental shelf he would know that he was close to 200 sea miles from home and could anticipate that within 48 hours he would lie snugly moored alongside the wharf in Nantucket harbor.

By the early 1800s New England whalers had rounded Cape Horn and entered the Pacific, -- home to another race of expert mariners. Guided by the sun and the stars, by the nature of the ocean swells, and by the navigational lore of generations of their ancestors, these storied navigators of the Pacific, confidently sailed their outrigger canoes, over vast distances across their vast ocean. No other people had ever matched the navigational skills of these Pacific wanderers, except perhaps for a people who lived10,000 miles away on the other side of the globe; -- the Vikings.

A reliable method of establishing Longitude had for long eluded mariners.

A disaster involving a Royal Navy fleet in 1707 was instrumental in again focusing attention on this problem.

A squadron of ships was returning north to England from the Mediterranean in foul weather, and when steering to enter the English Channel, ran aground on the Scilly Islands off the coast of Cornwall with the loss of 2000 lives.

Their navigation had been mostly by dead reckoning with only the occasional chance to confirm their latitude by sun sights.

The entrance to the English Channel is marked by the island of Ushant off the French coast to the south, and the Scilly Islands off Cornwall to the north. In 1707 both islands had primitive lighthouses. The entrance to the Channel is about 100 nautical miles wide. When the admiral in charge, on a northeasterly course to enter the Channel, sighted St. Agnes Lighthouse on the most westerly of the Scilly Islands off the coast of Cornwall, he mistook it for the Ushant lighthouse off the coast of France. The Ushant lighthouse was already 100 miles off to the south east, and the fleet was already north of the entrance to the Channel, wind and fate driving them blindly on to ruin and death on the treacherous rocks and reefs off the Scilly Islands. Sinister rumors had always abounded regarding that part of the coast of Cornwall, and the Scilly Islands in particular. Supposedly the good folk of those parts, in search of easy plunder, would light fires in strategic places to lure ships onto the rocks. There were even stories of lighthouse keepers turning off their lights in a storm, -- in the hope of reaping the bounty of the sea! Interestingly the Governor of the Scilly Islands had unabashedly objected to the construction of that same St Agnes Lighthouse, on the grounds that its construction would result in the loss of income from wrecks! However the disaster of 1707 was clearly due to a shipboard navigational error, and unrelated to any shore-bound skullduggery.

This would have been mainly an error in Latitude,-- an error of one degree, 25 minutes (85 nautical miles too far north), but it also represented an error in longitude as the Ushant lighthouse was one degree twenty minutes farther east than was the Scilly Island lighthouse -- at those latitudes an east/west error of 50 nautical miles.

This maritime disaster of the Royal Navy in home waters prompted a renewed interest in finding a solution to the "longitude problem" and a substantial reward was offered to anyone who could come up with a solution.

Prior to accurate chronometers a ship approaching a large land mass from the east or the west in poor visibility relied on dead reckoning and also on "coming into soundings."

Approaching the confines of the continental shelf, the ocean depth decreases from thousands of fathoms in the open ocean to less than one hundred fathoms approaching land on the continental shelf.

Since accurate Longitude is a function of accurate time, commencing in 1735, John Harrison began work on a portable clock that would keep accurate time despite the rigors of a sea voyage. For the next 30 years or so he labored to perfect his seagoing clock. Other methods such as Lunar distance and moons of Jupiter, were put forward, but these proved more cumbersome, requiring tedious and lengthy calculations. Eventually Harrison's chronometer became the recognized method for finding longitude.

The quest for longitude was not new. Around 150 b.c. a Greek astronomer, Hipparchus, working on the problem adopted Rhodes as a zero reference.

Amerigo Vespucci is reputed to have used lunar distance as a method of establishing his Longitude as early as the year 1499.

Also Magellan's pilot, San Martin in 1519 attempted, with some success, to establish Longitude using lunar distance.

Primitive instruments, and imprecise data relating to the positions and movements of celestial bodies, especially the moon, would have made such calculations, extremely difficult, and extremely unreliable, in those days.

It was this same explorer, Amerigo Vespucci who, when exploring down the east coast of what would later be known as South America, came to suspect that another entire continent intervened between him and China.

And it was the explorer, Balboa, who crossed the isthmus of Panama, and standing *"..upon a peak in Darien",* realized that an ocean also intervened!

And it was the poet, Keats who had the wrong explorer in mind when he spoke of *"..stout Cortez, when with eagle eyes he star'd at the Pacific…silent upon a peak in Darien."!*

Galileo considered using four of Jupiter's moons, Io Europa, Ganymede and Callisto, as a universal clock. These four moons, visible with even a weak telescope, act like the hands of a cosmic clock.

Captain Cook setting out on his second voyage in 1772 took two of the new chronometers with him and expressed himself delighted with their accuracy, and with this newfound ability to easily and accurately determine his longitude. On a prior voyage, using the more tedious Lunar distance method, Cook, a man with minimal formal education, had charted the coast of New Zealand with remarkable accuracy. On all of his voyages Cook had the benefit of the relatively newly minted Nautical Almanac, which would have greatly facilitated his Lunar Distance calculations. Most purists, I'd imagine, would retain a place in their hearts for the totally self contained Lunar Distant method of determining Greenwich Mean Time. Chronometers while convenient, could fail.

Most ships that could afford them carried more than one chronometer, and usually more than two, as any time difference between two chronometers would not determine which was in error.

Darwin's Beagle carried 22 chronometers! However the Beagle's main mission was surveying, and the Beagle's voyage was financed by the British Government,

Chronometers were kept in a special compartment, cosseted amidships so as to minimize the effect of ship movements. A hack, or deck watch, synchronized as needed to the chronometer would be used on deck or ashore to assure accurate Greenwich Mean Time (GMT). The chronometer, a valued instrument, would be protected from any harsh environment to the extent possible.

While the chronometer gradually superseded Lunar Distance as the preferred method for establishing Longitude, this latter method did not entirely fade into obscurity.

"Lunars" were still useful as a means of checking the accuracy of the chronometer, -- and were frequently used for that purpose.

Here is a brief description on how Lunar Distance is utilized to establish Greenwich Mean Time (GMT)

The moon moves at a rate of 0.5 degrees per hour, as seen against the background of the stars.

This relatively rapid movement of the moon can be a useful measurement of accurate time.

To measure Lunar Distance, the angle between the moon and another selected celestial body is measured. For technical reasons horizontal, rather than vertical separation, is preferred.

The celestial bodies used for Lunar Distance measurements are the sun, the planets, and some selected bright stars close to the moon's orbit.

Applying corrections for Parallax, Refraction and Semi diameter, this angle is then corrected (cleared) to equate it to the angle one would get from the center of the earth, -- referred to as a geocentric angle.

Since it is somewhat inconvenient to take a sighting from the center of the earth!, the angle obtained on the surface is corrected "cleared" to equate it to the angle taken at the center of the earth.

This angle, as measured from the center of the earth in, let us say the South Pacific, will be the same angle as measured from the center of the earth at Greenwich, *at that same time.*

The times at which these same angular distances between the moon and these selected celestial bodies takes place at Greenwich can be found in various precomputed tables. Formerly these times were listed in the Nautical Almanac from its first printing in 1766, up until 1906.

In general then the steps are:

--Measure the angle between the moon and the selected celestial body.

--Measure the altitude of the moon and the altitude of the celestial body. These altitude measurements are necessary to determine corrections for Parallax and Refraction.

--By applying corrections for Parallax, Refraction and Semi Diameter of the moon, we correct (clear) the angle between the Moon and the Celestial body to a Geocentric angle, i.e. an angle at the center of the earth.)

-- From the precomputed tables determine the time, at Greenwich, at which the Moon and that particular selected celestial body would be separated from each other by that same geocentric angular distance, as measured at the observer's position.

-- This gives us the time at Greenwich, the Prime Meridian, a prerequisite for establishing our Longitude.

Prior to 1767, and the first publication of the Nautical Almanac, Navigators would have had to involve themselves in a far more rigorous process in establishing Longitude by Lunar Distance.

Among other meticulous calculations they would have had to ascertain the (positions) Declination and Right Ascension of both the Moon and the selected Celestial body, for an assumed Greenwich Mean Time (GMT)

Establishing the moon's position alone, for a specific time, would have been a particularly tedius and time consuming exercise, involving complex formulae, and a knowledge of advanced mathematics.

First, as above, they would measure the lunar distance, together with the sextant altitude of the moon and the selected celestial body.

Then they would reduce (clear) this measured lunar distance angle to a geocentic angle.

Next they would compute, mathematically the lunar distance angle for an assumed Greenwich Mean Time (GMT). This would involve manually solving a spherical triangle whose apices would be the observer, the moon, and the selected celestial body for that particular assumed time.

Solving this spherical triangle, using the spherical law of cosines,

https://en.wikipedia.org/wiki/Spherical_law_of_cosines

would provide them with a computed geocentric lunar distance for that specific assumed Greenwich Mean Time (GMT)

They would then compare this computed lunar distance for that assumed time, to their actual observed lunar distance. Any difference in distance would correspond to a difference in time.

Let's say that the computed geocentric lunar distance for an assumed time of 1600 GMT, was 25 degrees 30 minutes, and the actual measured geocentric lunar distance was 25 degrees 45 minutes.

This would equate to a difference between observed and computed lunar distances of 15 degrees.

The moon travels, relative to the background stars, at 0.5 degrees per hour, or 1 minute of arc for every 2 minutes of time. Therefore the difference in lunar distance (15 minutes of arc) between observed and computed lunar distances would correspond to an added time difference of 30 minutes. The time difference would be added since the observed lunar distance is larger.

Thus the GMT for our actual observed lunar distance of 25 degrees 45 minutes would be 1630 GMT.

Such calculations, prior to precomputed tables, would have taken hours.

Above we are using an assumed time as a reference, which enables us to establish our actual time. Compare this to the intercept method of plotting a line of position, where we use an assumed position as a reference, which enables us to establish our actual position.

It's possible that both Saint-Hilaire and Sumner may themselves have stood on the shoulders of earlier mariners.

Most likely the concept of using an assumed position in order to plot an actual line of position, was borrowed from the concept of using an assumed time in Lunar Distance calculations.

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David Thompson exploring Canada's north west, in the late 1700s relied solely on the Lunar Distance method of determining longitude. In that era he would have had the convenience of a Nautical Almanac with precomputed tables. He believed that a chronometer would be vulnerable in the harsh land environment of his exploration. A mariner, accustomed to storms, and an oftentimes harsh environment at sea, might find himself bemused by such reasoning!

The above is meant merely as a basic introduction to the principles of Celestial Navigation, and I apologize for its lengthiness, its repetitiveness, --- and its diversions.

The actual procedural details of taking a sight, of applying sextant corrections for dip, refraction and other corrections, together with the use of the Nautical Almanac and Sight Reduction tables, while requiring meticulous attention to detail, are procedural details that become routine with practice. Various "*Sight reduction work sheets"* also exist to help guide the navigator methodically through the process.

The more important thing is to understand the concept. This is not, however, to underestimate the importance of attention to precise detail in sight reduction, where a seemingly small procedural error can place the seafarer in a wheat field in Saskatchewan, instead of eagerly anticipating landfall off the Hawaiian islands!

Attempting to explain celestial navigation without direct instructor/student interaction can be akin to attempting to describe a spiral staircase while sitting on ones hands!

A good two part presentation, by a Captain Will Lesh, explaining celestial navigation is:

https://www.youtube.com/watch?v=-ARXW8InStY

Another more comprehensive resource with multiple episodes, each dedicated to a specific aspect of Celestial navigation is:

https://www.youtube.com/watch?v=hDd1es5oQto

Lastly is the three part series, enlivened by cartoon characters, and beloved by Air Force navigation instructors:

https://www.youtube.com/watch?v=cun0DGZ6-sk

▼ The answer to question on page 56 is:
No.
--If you were on the bearing line from your actual position to the ground position (GP), the intersection of such a line with the circle of equal altitude for Ho would indeed constitute a two position line navigational fix.
However you are not on that line.
--Remember that the bearing; assumed position (AP) to the ground position (GP), is from an assumed position. This bearing, while usually close, is not likely to coincide

exactly with the actual bearing from your real position. Therefore you are not necessarily on the bearing line, AP to GP.
--While you are somewhere on the circle of equal altitude for Ho, which (in this rare instance) happens to run through the assumed position (AP), you are unlikely to be at the assumed position (AP) itself. All you can infer is that you are somewhere on a line of position (LOP) running through, (in this instance), the assumed position (AP).
--Remember the basics. Anyone anywhere on that circle of equal altitude for Ho would get the same sextant altitude for that body for that time.
In this scenario, a single line of position (LOP) running through the assumed position (AP) is obtainable, (Diagram page 56) but without a second line of position (LOP) to intersect this first, a navigational fix is not.

For having had the interest, -- and the patience -- to stay with me to this point in the tutorial, I thank you.

Denis Rogers.
RCAF Navigator in the 1950s, -- in the twilight of celestial navigation in the air.